Detection of excited state quantum phase transition in squeezed Kerr-nonlinear resonators

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Francisco Pérez-Bernal, and Lea F. Santos

Platform for detecting ESQPT

Squeezed Kerr oscillator: platform to detect ESQPTs

FREQUENCY & TIME domain measurements

Platform for detecting ESQPT

Squeezed Kerr oscillator: platform to detect ESQPTs

FREQUENCY & TIME domain measurements

- **Spectrum**

The system exhibits basic feature of ESQPT

Vanishing of energy gaps

Divergence of the DOS

- **Quantum dynamics**

Consequences of the presence of the ESQPT

Exponential growth of OTOCs
(scrambling – instability)

Slow evolution of survival probability
(localization)

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Caprio, Cejnar, Iachello
Ann. Phys. **323**, 1106 (2008)

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PRE **101**, 010202(R) (2020)

Slow evolution of the survival probability
(localization) $|\langle\Psi(0)|\Psi(t)\rangle|^2$

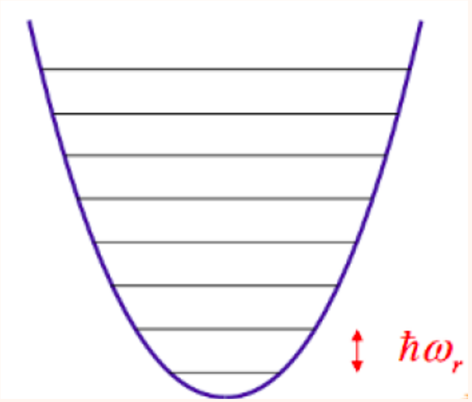
PRA **92,** 050101R (2015)
PRA **94**, 012113 (2016)
Fortschr. Phys. (2017)

Curro Pérez-Bernal
Lea F. Santos

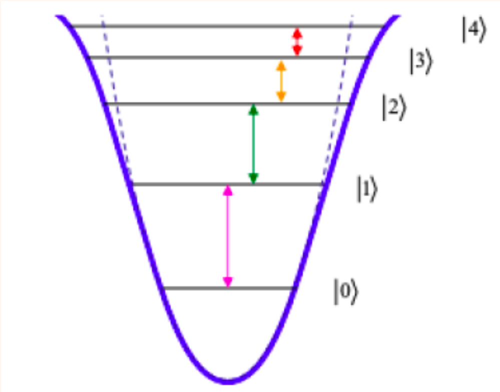
Hamiltonian

Squeezed Kerr oscillator $\hat{H} = -K\hat{a}^{\dagger 2}\hat{a}^2 + \epsilon_2(\hat{a}^{\dagger 2} + \hat{a}^2)$

$$\hat{n}(\hat{n} - 1)$$



Harmonic oscillator

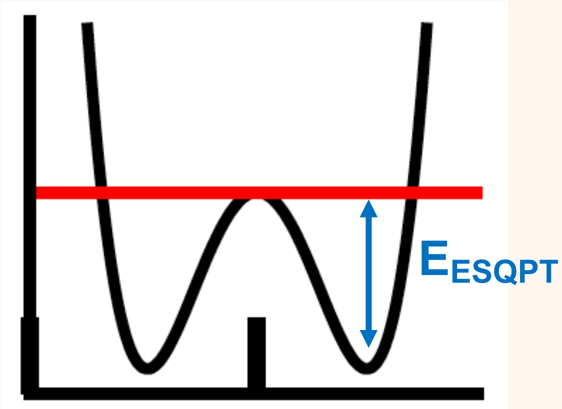


Anharmonic oscillator
Selective transitions between energy levels

Hamiltonian

Squeezed Kerr oscillator $$\hat{H} = -K\hat{a}^{\dagger 2}\hat{a}^2 + \epsilon_2(\hat{a}^{\dagger 2} + \hat{a}^2)$$ $\hbar_{\text{eff}} = 1$

$\xi = \epsilon_2/K$ Control parameter
(competition between 2 terms)



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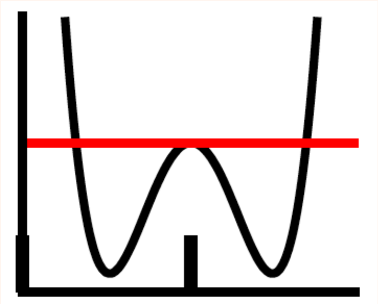


K $= K/(2\pi)$

$$\hat{H}_{qu} = \hbar_{\text{eff}}^2 \mathsf{K}\hat{n}(\hat{n}-1) - \hbar_{\text{eff}}\mathsf{K}\xi\left(\hat{a}^{\dagger 2} + \hat{a}^2\right)$$

$\hbar_{\text{eff}}$ to scale the Hamiltonian

$$\hbar_{\text{eff}} \to 0$$

(classical limit)

Vanishing of Energy Gaps

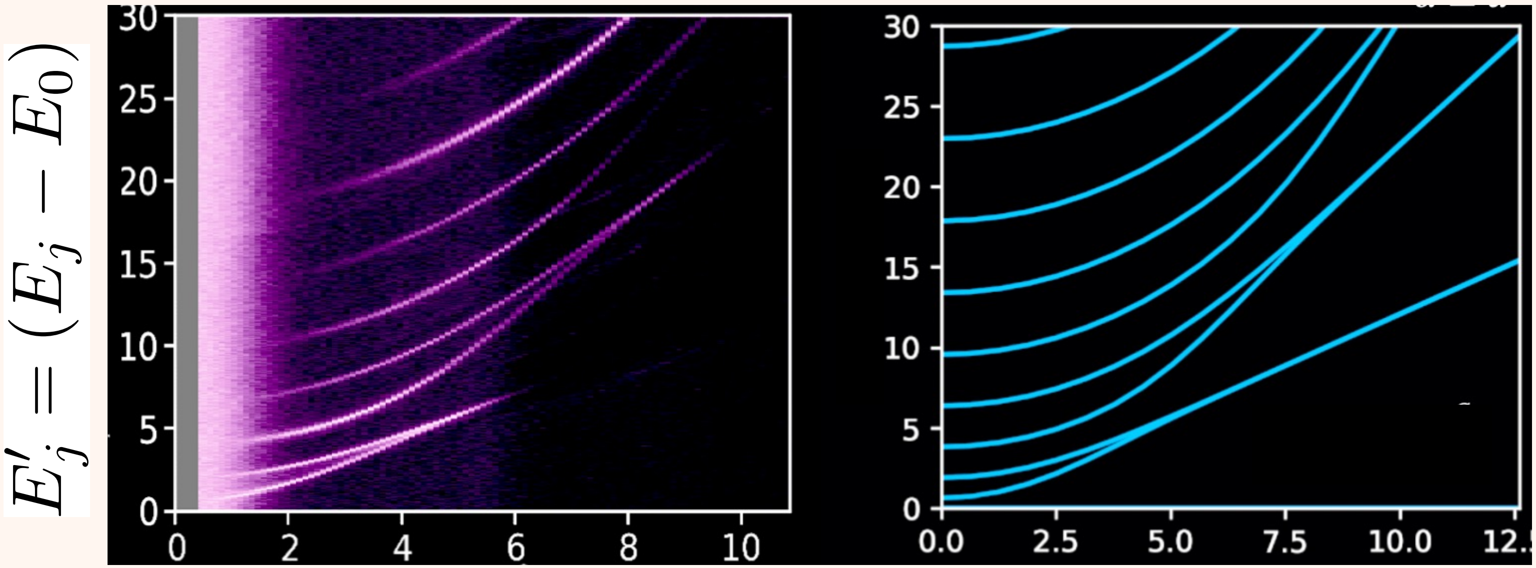
$$\hat{H}_{qu} = \hbar_{\text{eff}}^2 \mathsf{K} \hat{n}(\hat{n} - 1) - \hbar_{\text{eff}} \mathsf{K} \xi \left(\hat{a}^{\dagger 2} + \hat{a}^2 \right)$$

$\mathsf{K} = 0.319\,\text{MHz}$

ξ control parameter

$\hbar_{\text{eff}} = 1$

EXPERIMENT THEORY



ξ ξ

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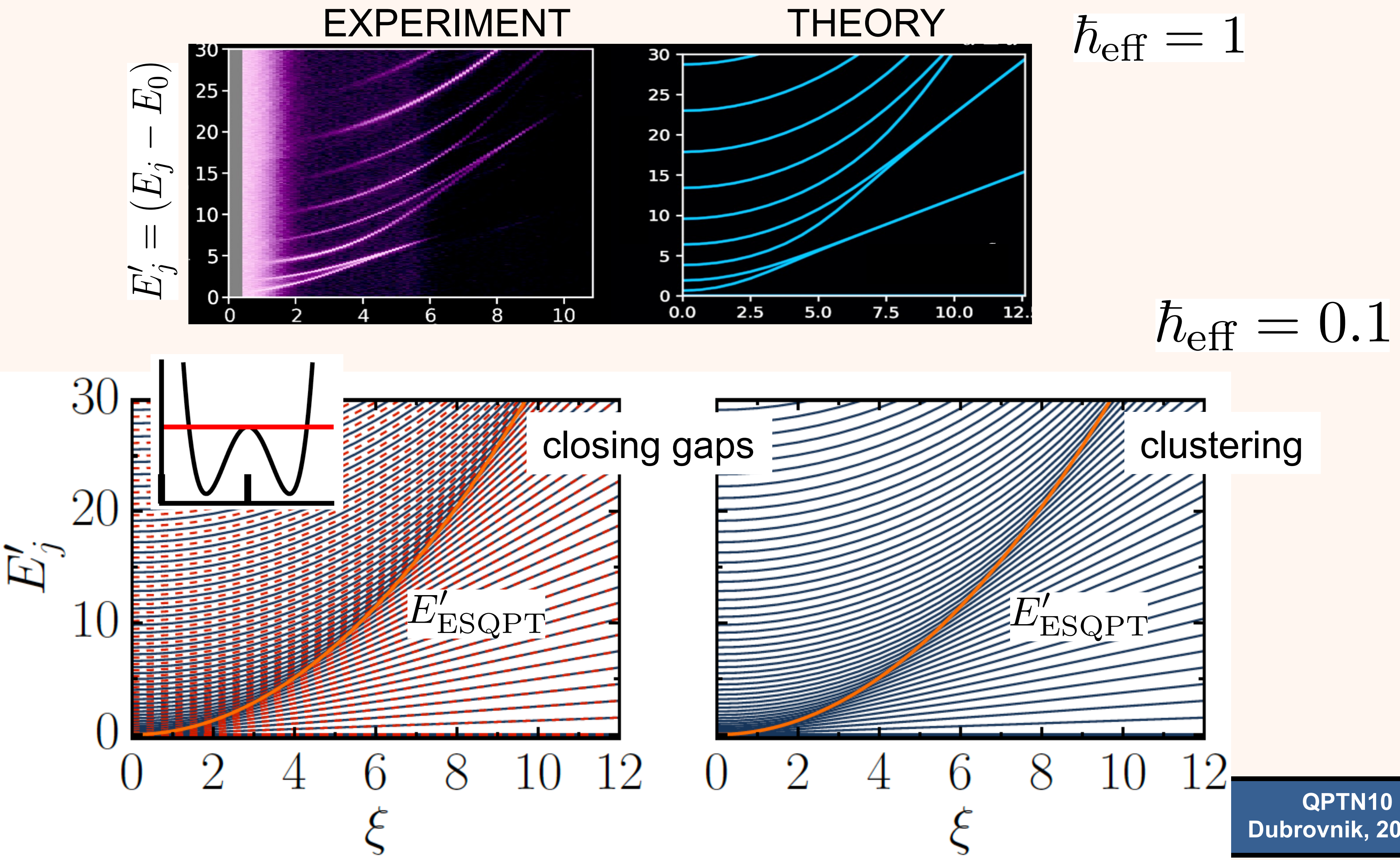
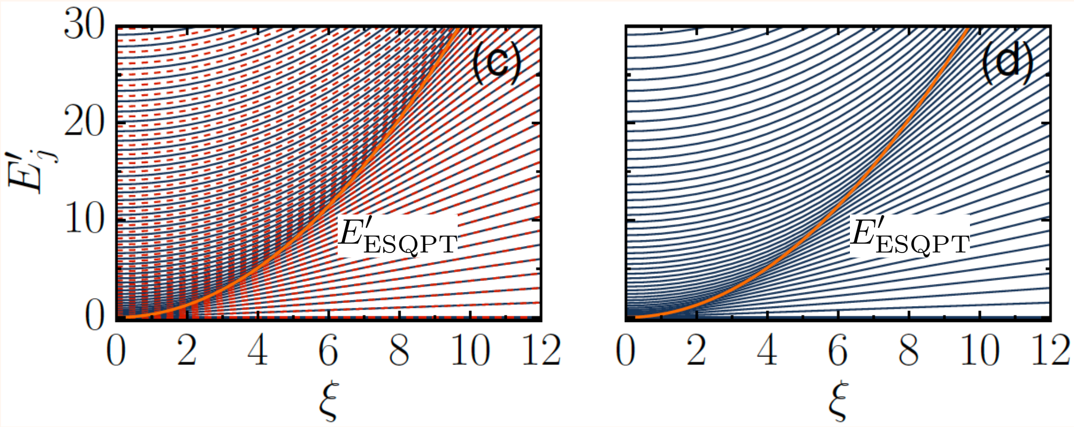
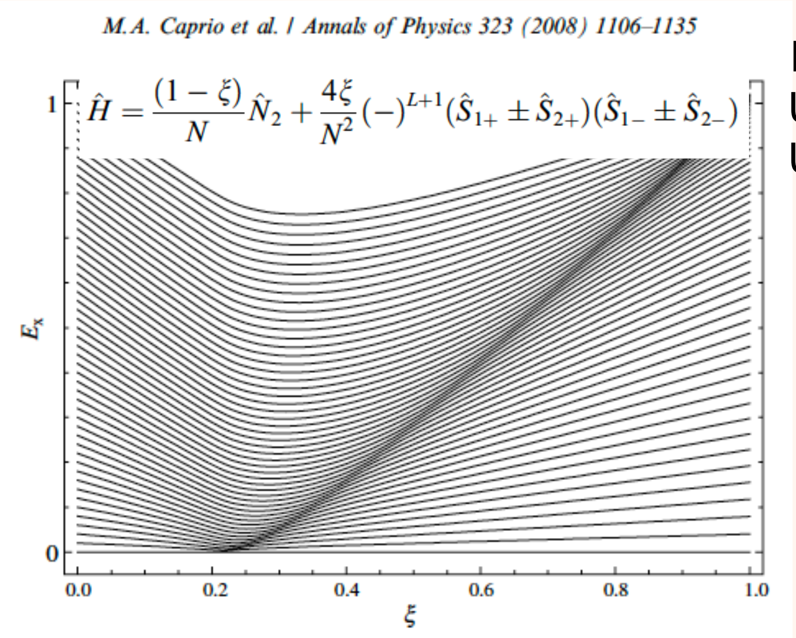
Vanishing of Energy Gaps

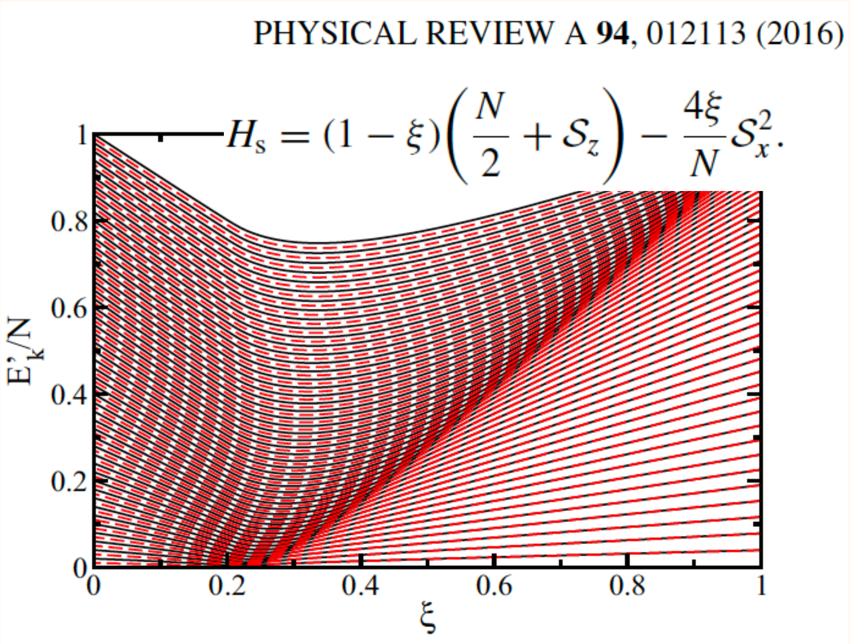


Vanishing of Energy Gaps

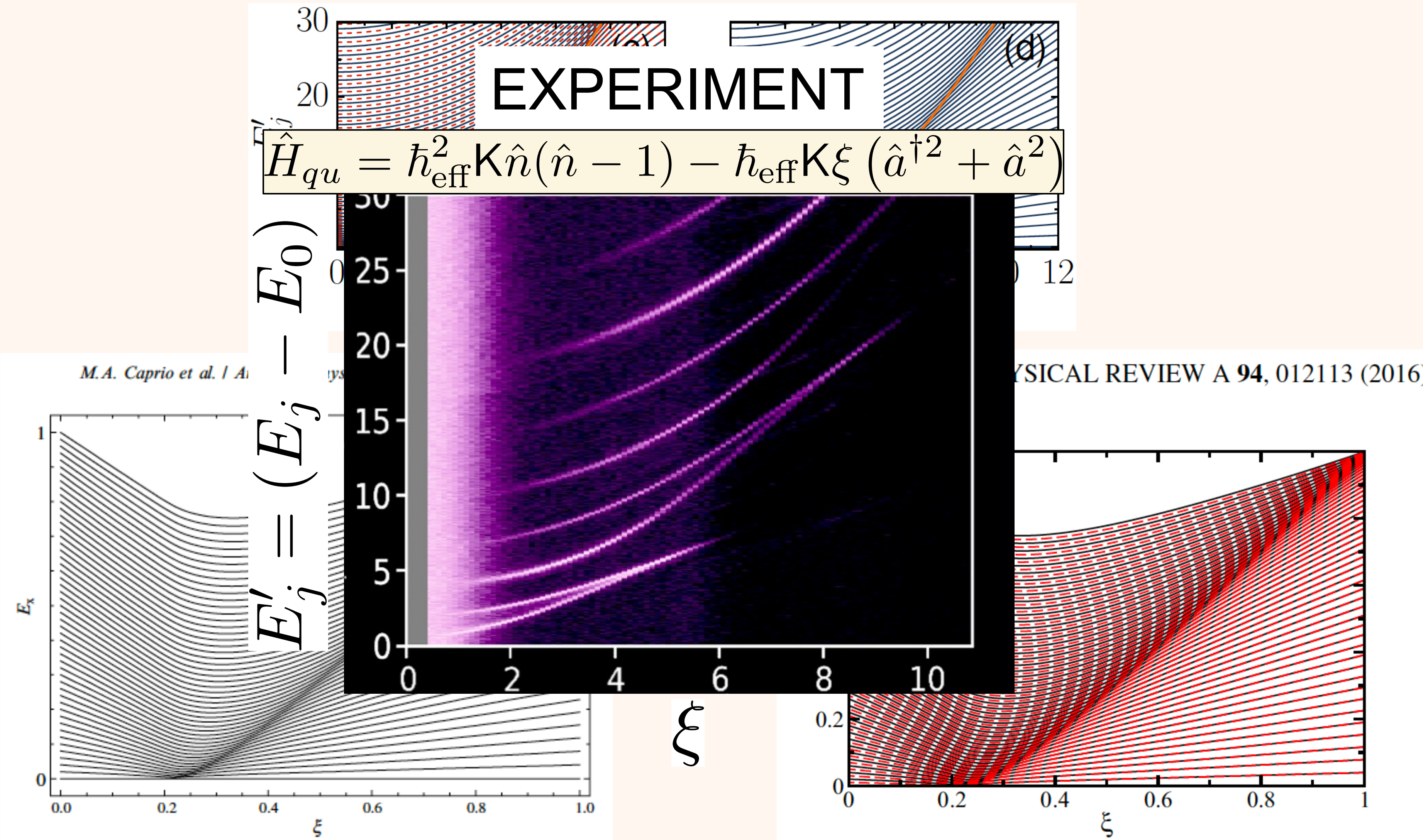




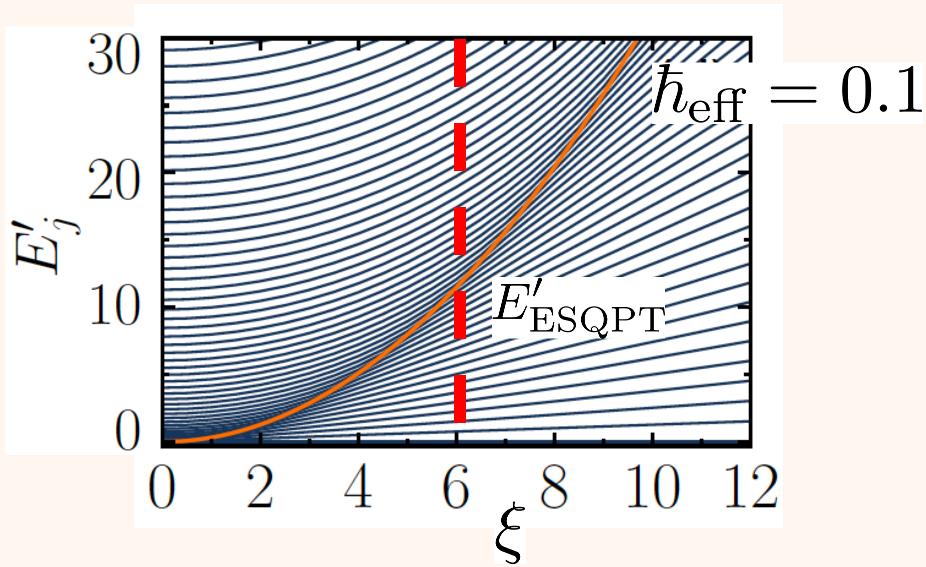
Lipkin U(2)
U(3) vibron
U(4) vibron
U(6) IBM
etc



Vanishing of Energy Gaps



Clustering of the Eigenvalues
Divergence of DOS

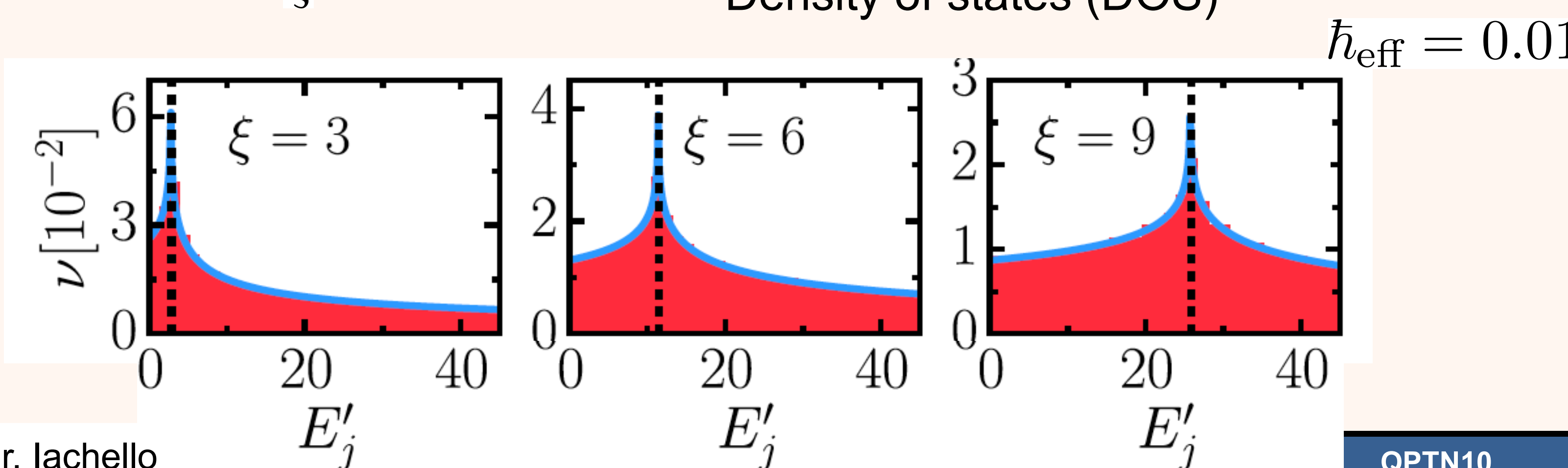


Hamiltonian is unbounded - Fock

Truncated N ensures convergence of the levels

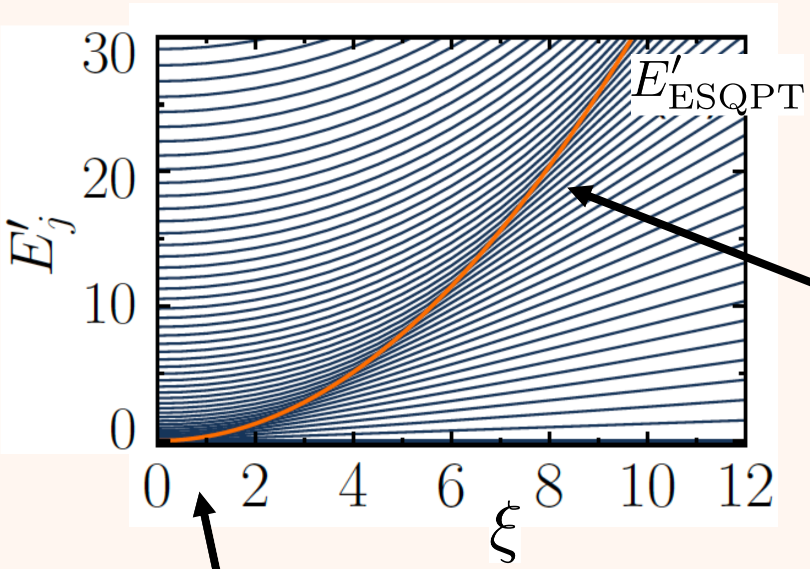
1 effective degree of freedom

Density of states (DOS)



Caprio, Cejnar, Iachello
Ann. Phys. **323**, 1106 (2008)

Localization at the ESQPT



$$|\psi\rangle_{\rm ESQPT} \sim |0\rangle$$

$\xi = 0$

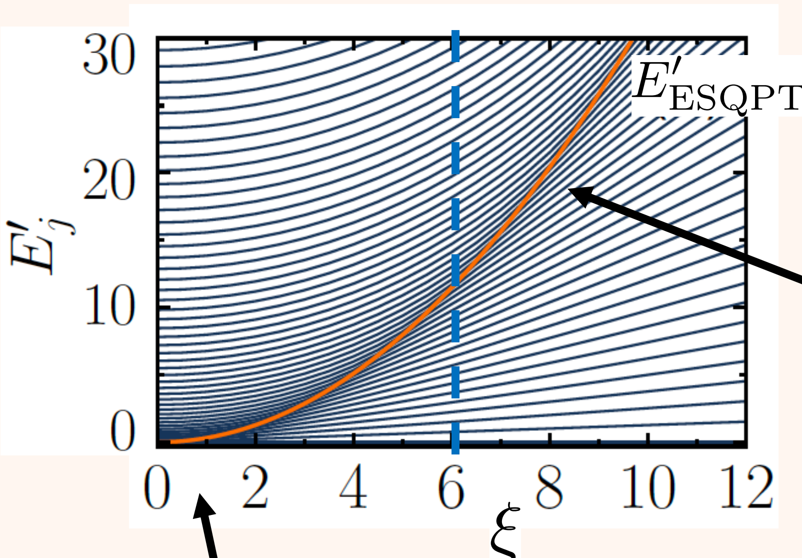
$$|\psi_{\rm GS}\rangle = |0\rangle$$

Bounded Hamiltonians

PRA **92,** 050101R (2015)
PRA **94**, 012113 (2016)
Fortschr. Phys. (2017)

$$\hat{H}_{qu} = \hbar_{\rm eff}^2 \mathsf{K} \hat{n}(\hat{n}-1) - \hbar_{\rm eff} \mathsf{K} \xi \left(\hat{a}^{\dagger 2} + \hat{a}^2\right)$$

Localization at the ESQPT



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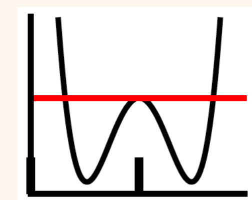
PRA **92,** 050101R (2015)
PRA **94**, 012113 (2016)
Fortschr. Phys. (2017)

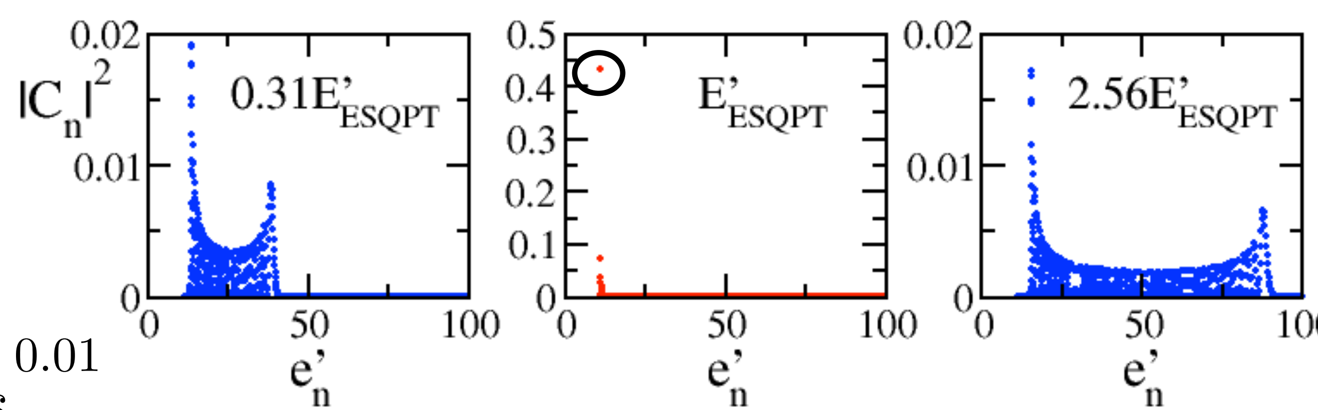


$$|\psi\rangle_{\mathrm{ESQPT}} \sim |0\rangle$$

$$\xi = 0$$

$$|\psi_{\mathrm{GS}}\rangle = |0\rangle$$

$$|\psi^{(k)}\rangle = \sum C_n^{(k)} |n\rangle$$ (Fock basis)



$\hbar_{\mathrm{eff}} = 0.01$
$\xi = 6$

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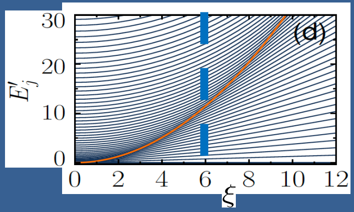
Localization at the ESQPT

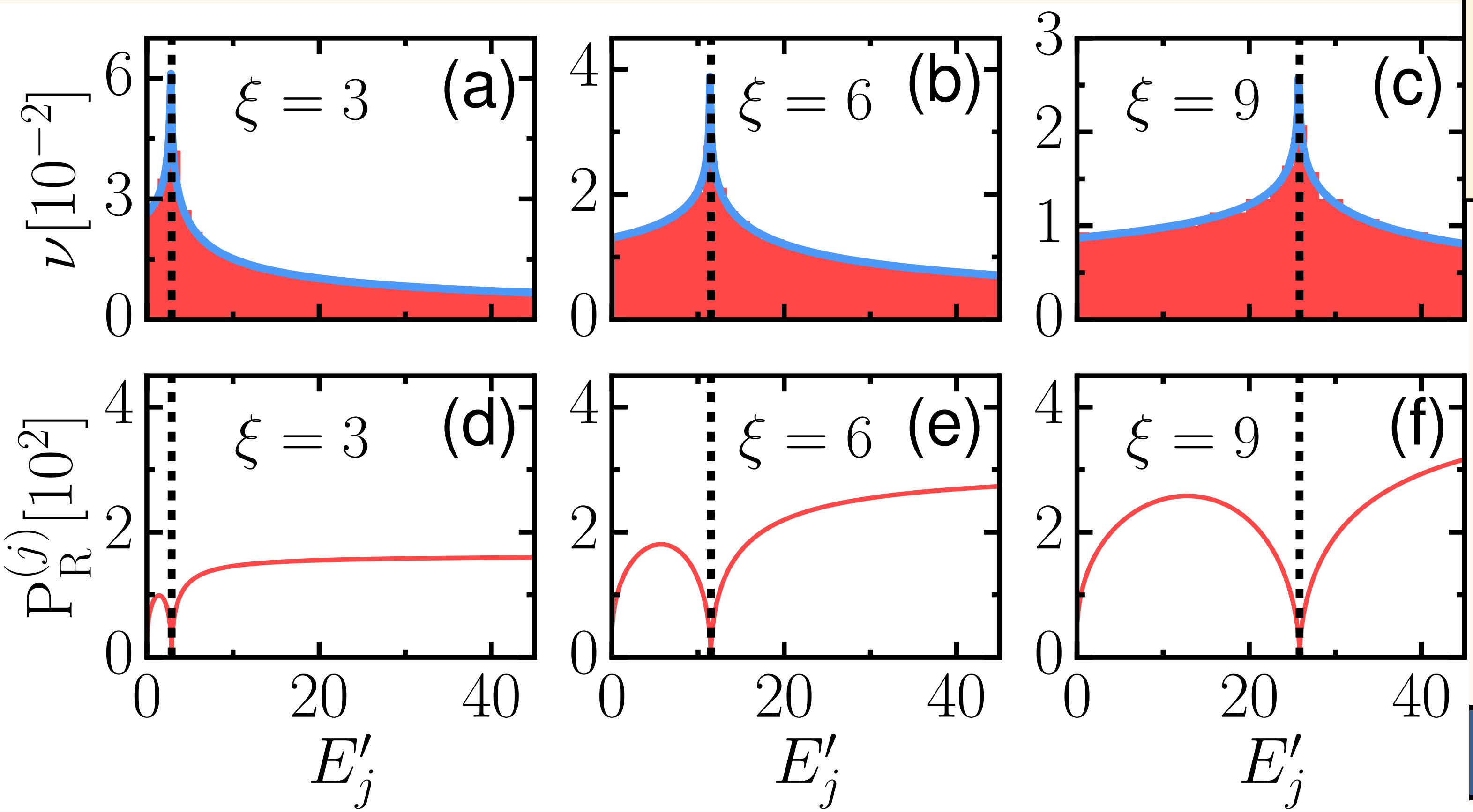


$$|\psi^{(k)}\rangle = \sum_n C_n^{(k)} |n\rangle \qquad \mathrm{P}_\mathrm{R}^{(k)} = \frac{1}{\sum_{n=0}^{N} |C_n^{(k)}|^4}$$

PRA **92**, 050101R (2015)
PRA **94**, 012113 (2016)
Fortschr. Phys. (2017)

Prague (2016)
QPTN 8

Padova (2018)
QPTN 9



$\hbar_{\rm eff} = 0.01$

Classical Limit of the Kerr Resonator

Quantum Hamiltonian

$$\hat{H}_{qu} = \hbar_{\text{eff}}^2 \mathsf{K} \hat{n}(\hat{n}-1) - \hbar_{\text{eff}} \mathsf{K} \xi \left(\hat{a}^{\dagger 2} + \hat{a}^2\right)$$

$$\hat{a} = \frac{\hat{q} + i\hat{p}}{\sqrt{2\hbar_{\text{eff}}}}$$

$$\hat{a}^\dagger = \frac{\hat{q} - i\hat{p}}{\sqrt{2\hbar_{\text{eff}}}}$$

$$\hbar_{\text{eff}} \to 0$$

Classical Hamiltonian

$$H_{cl} = \frac{\mathsf{K}}{4}(q^4 + p^4 + 2q^2p^2) - \mathsf{K}\xi(q^2 - p^2)$$

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Stránsky & Cejnar
PLA 380, 2637 (2016)

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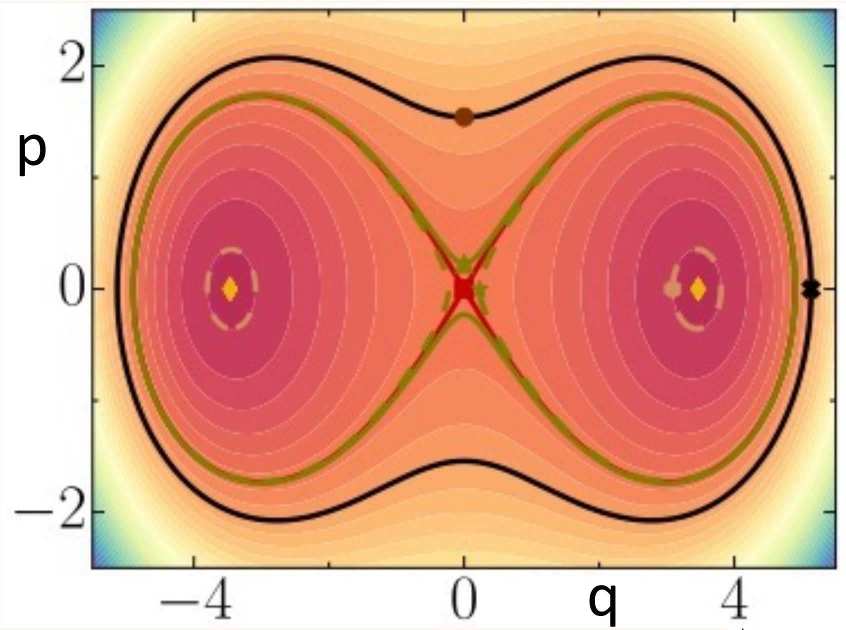
$$\hat{H}_{qu} = \hbar_{\text{eff}}^2 \mathsf{K} \hat{n}(\hat{n}-1) - \hbar_{\text{eff}} \mathsf{K}\xi \left(\hat{a}^{\dagger 2} + \hat{a}^2\right)$$



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$$\hbar_{\text{eff}} \to 0$$

Classical Hamiltonian

$$H_{cl} = \frac{\mathsf{K}}{4}(q^4 + p^4 + 2q^2p^2) - \mathsf{K}\xi(q^2 - p^2)$$

Critical points

Center points

$\{-\sqrt{2\xi}, 0\}$ $\{\sqrt{2\xi}, 0\}$

Hyperbolic point

$\{0, 0\}$

ESQPT Separatrix (Bernouilli Lemniscat)

$$E'_{\text{ESQPT}} = \mathsf{K}\xi^2$$

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Lea F. Santos

Stránsky & Cejnar
PLA 380, 2637 (2016)

Classical Limit of the Kerr Resonator

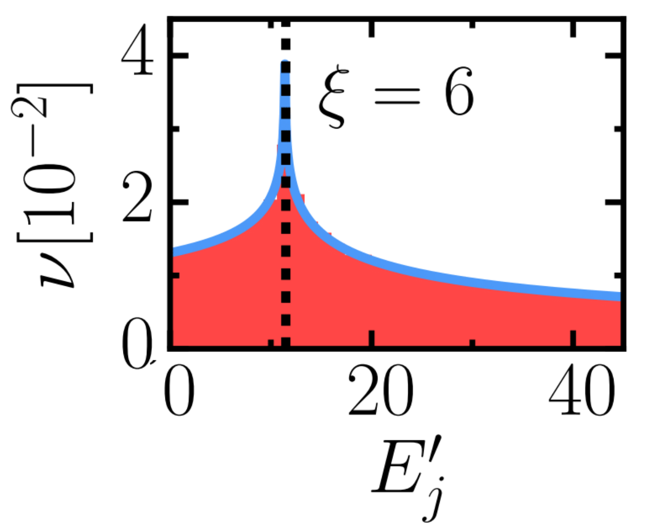
Density of states

Smooth component of Gutzwiller trace formula

$$\nu(\mathcal{E}) = \frac{1}{2\pi}\int dp\,dq\,\delta(H_{cl}-\mathcal{E})$$

$$\nu(\mathcal{E}) = \frac{1}{2\pi}\int_{q\in\Omega_{\mathcal{E}}} \frac{dq}{2\sqrt{(2\sqrt{\mathrm{K}\,u(\mathcal{E})}-(\lambda+\mathrm{K}q^2))u(\mathcal{E})}}$$

$u(\mathcal{E}) = \mathcal{E}-\mathcal{E}_{\min}+\lambda q^2$, $\lambda = 2\mathrm{K}\xi$



Husimi Quasidistribution

Squared overlap between a quantum state and the coherent state (Bargmann) representation of quantum states in phase space. PRE **65**, 036205 (2002)

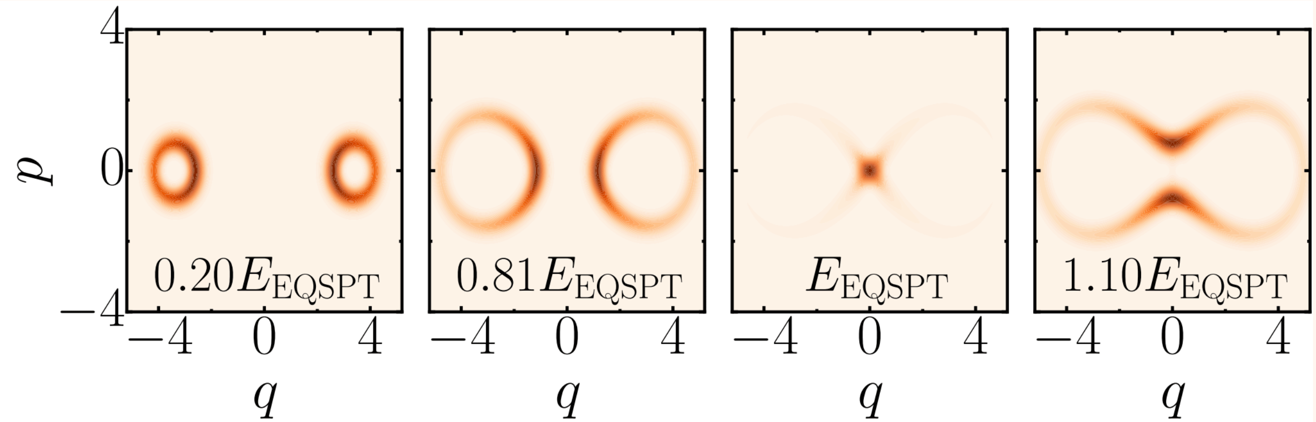
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad |\alpha\rangle = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^\dagger}|0\rangle \qquad \alpha = (q + ip)/\sqrt{2\hbar_{\text{eff}}}$$

$$Q_\psi(\alpha) = |\langle \alpha|\psi\rangle|^2$$

$$|\psi^{(j)}\rangle = \sum_n C_n^{(j)} |n\rangle \qquad Q_{\psi^{(j)}}(q,p) = \frac{1}{2\pi\hbar_{\text{eff}}} \left| \sum_{n=0}^{N} C_n^{(j)} e^{-\frac{q^2+p^2}{4\hbar_{\text{eff}}}} \frac{(q-ip)^n}{\sqrt{2^n \hbar_{\text{eff}}^n n!}} \right|^2$$

$$\hbar_{\text{eff}} = 0.1$$

$$\xi = 6$$



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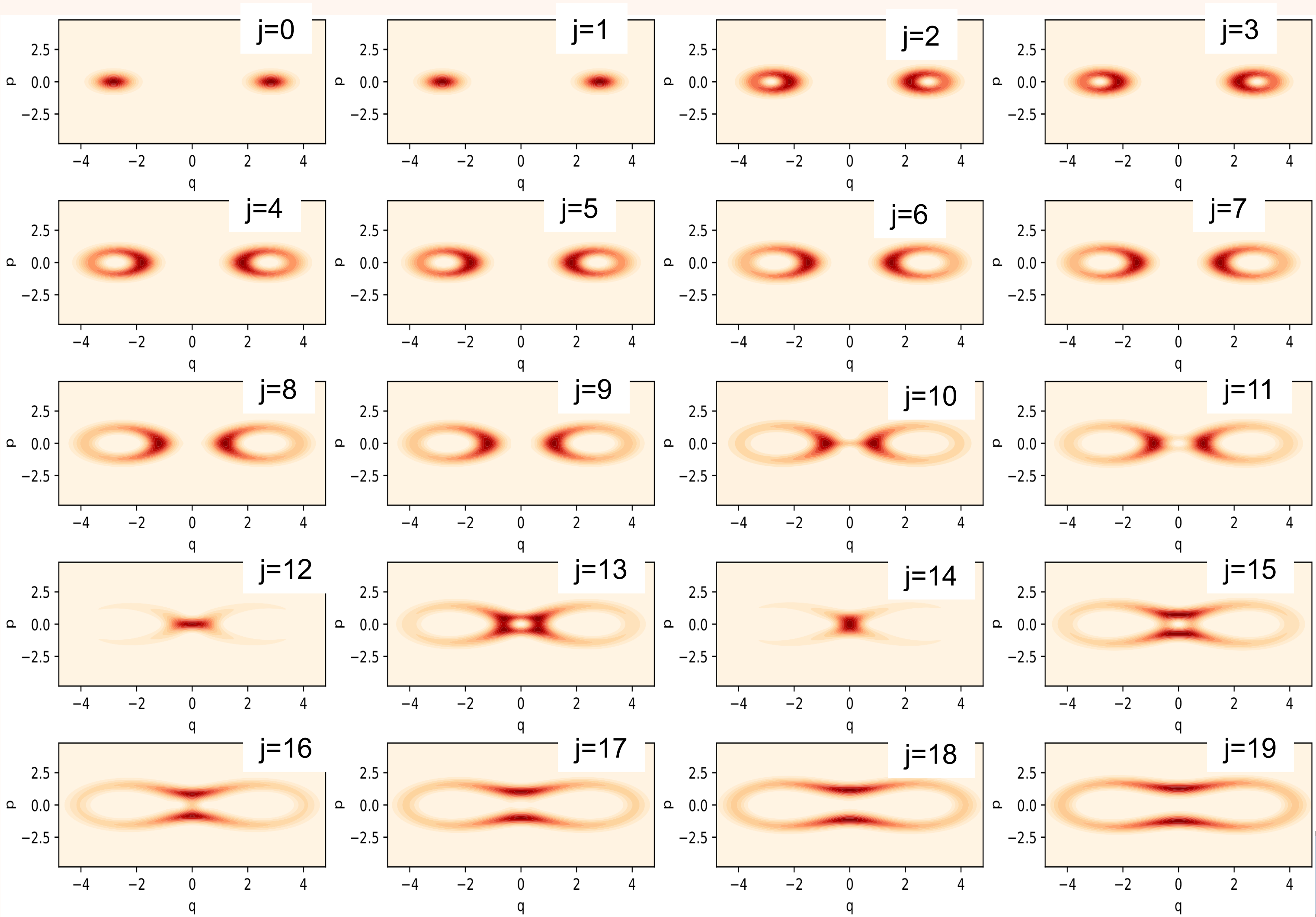
Husimi Quasidistribution

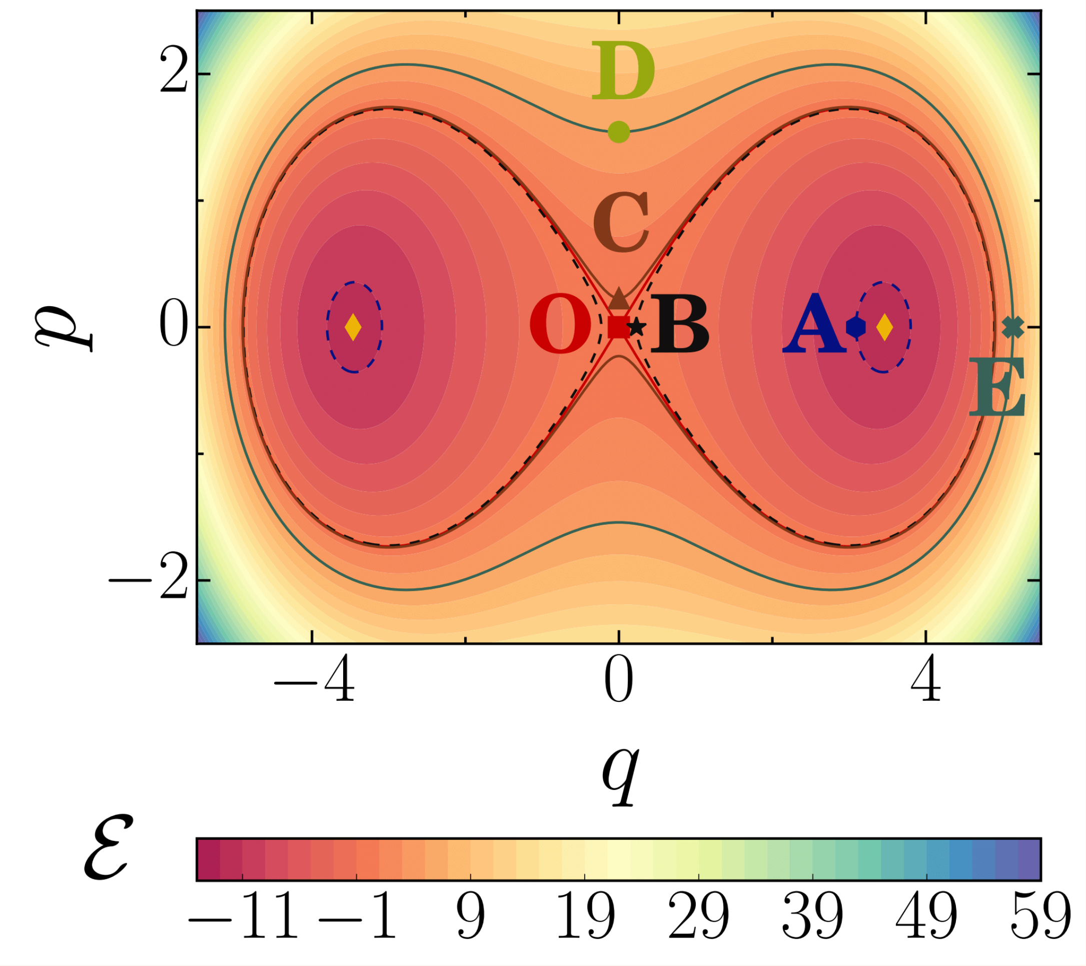


$\xi = 4$

$\hbar_{\text{eff}} = 0.2$

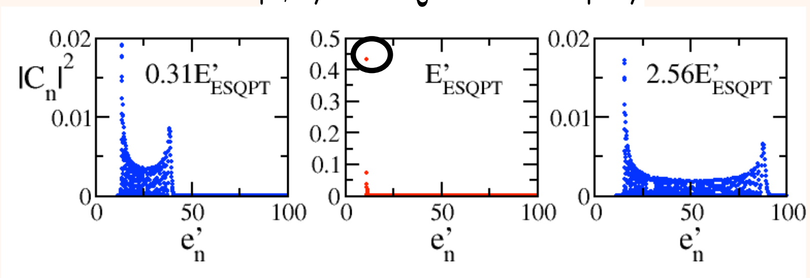
Classical Limit of the Kerr Resonator

Selected initial conditions and associated orbits in phase space

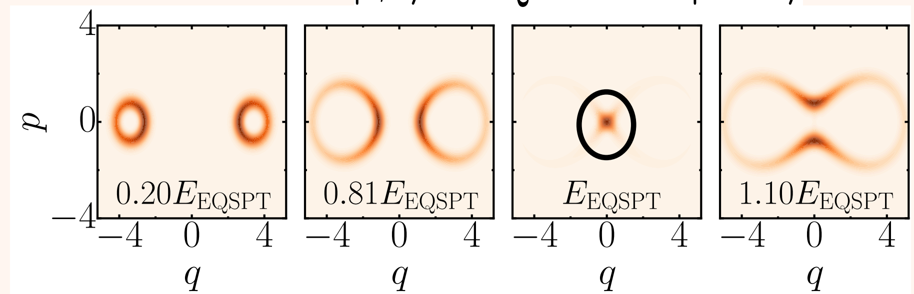


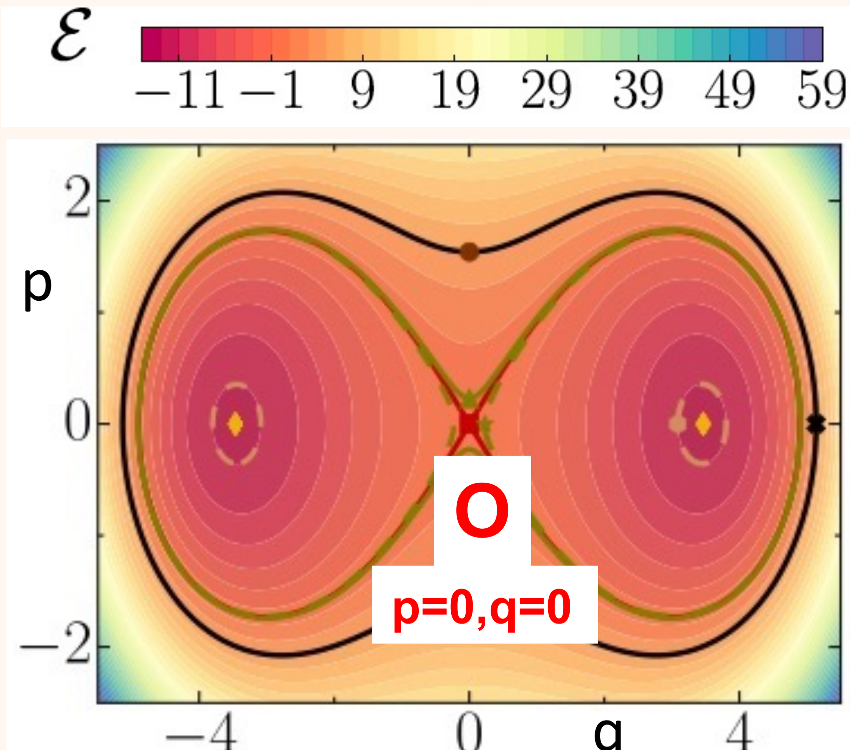
Survival Probability: Slow Evolution for O

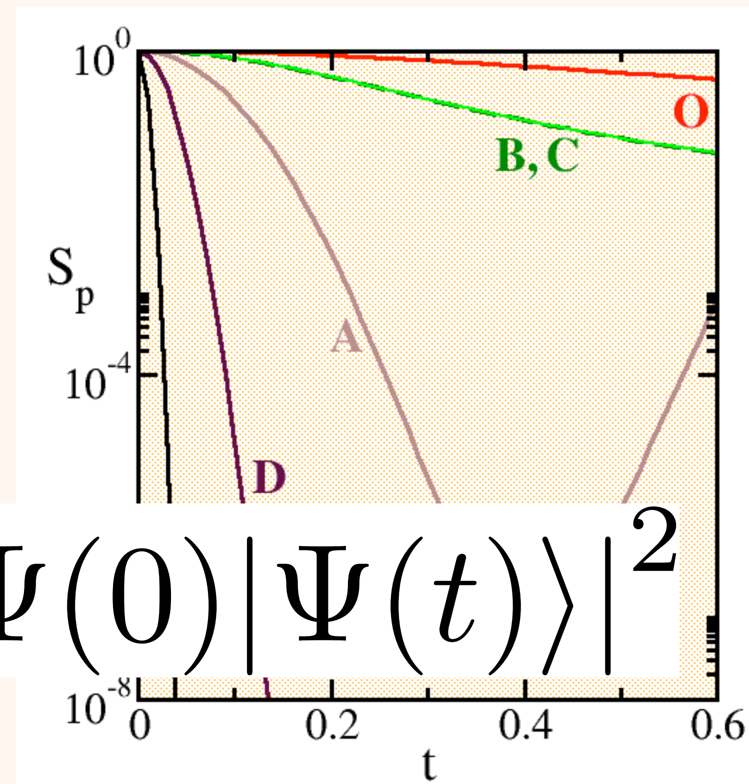
$|\psi\rangle_{\text{ESQPT}} \sim |0\rangle$



$|\psi\rangle_{\text{ESQPT}} = |\alpha_O\rangle$



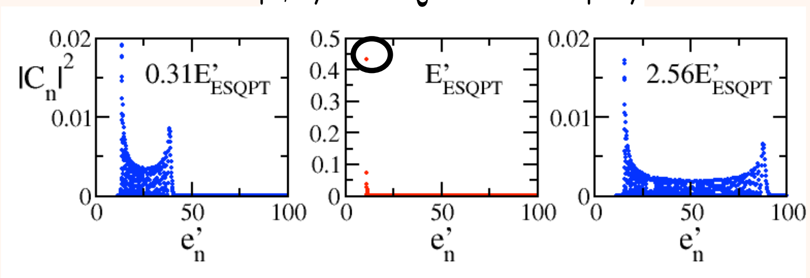




$$|\langle\Psi(0)|\Psi(t)\rangle|^2$$

Survival Probability: O vs A-E

$|\psi\rangle_{\mathrm{ESQPT}} \sim |0\rangle$



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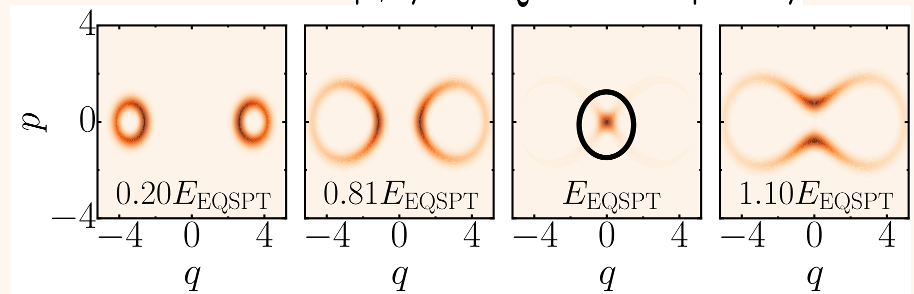
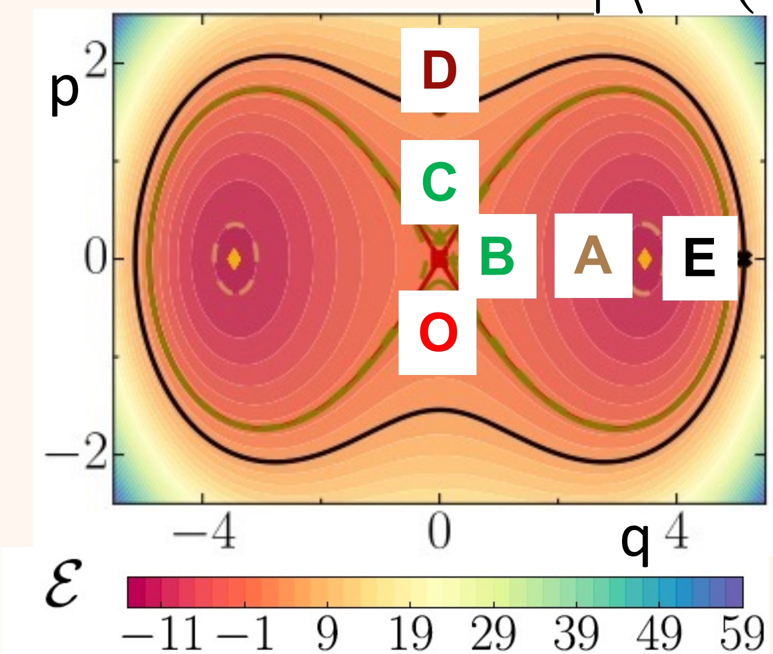
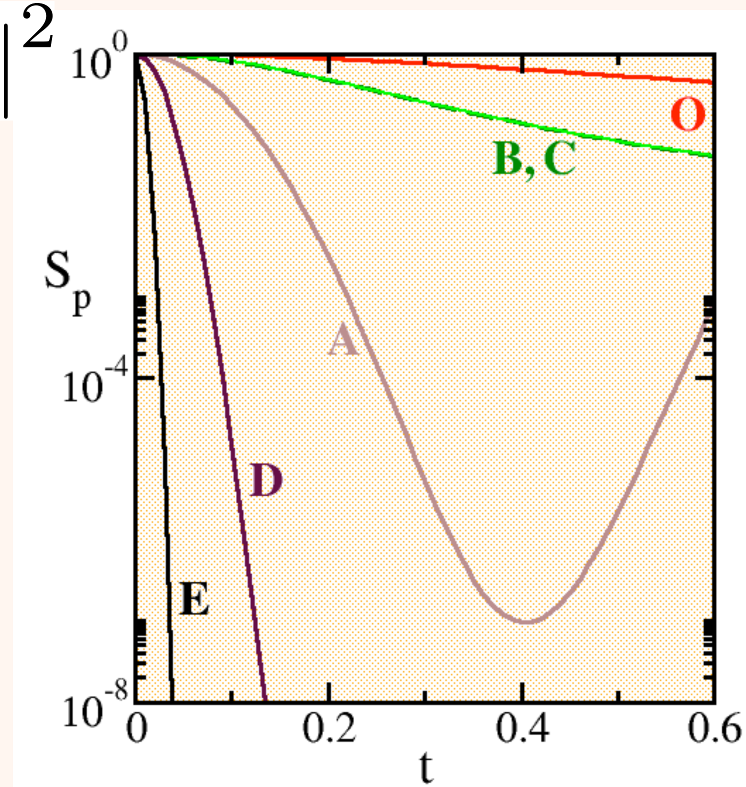
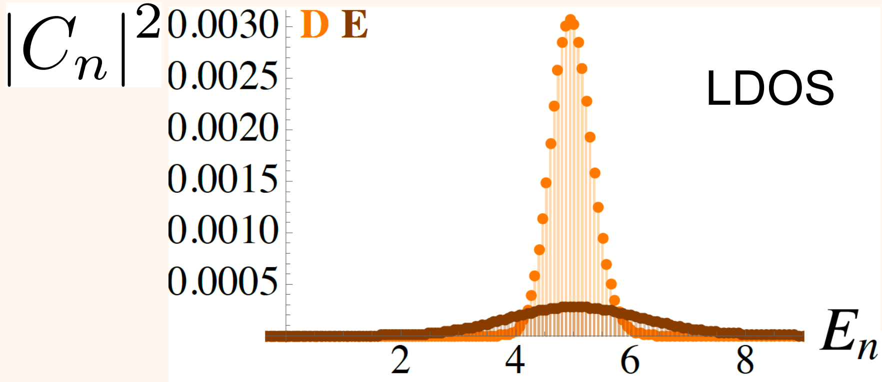


$$|\langle\Psi(0)|\Psi(t)\rangle|^2$$

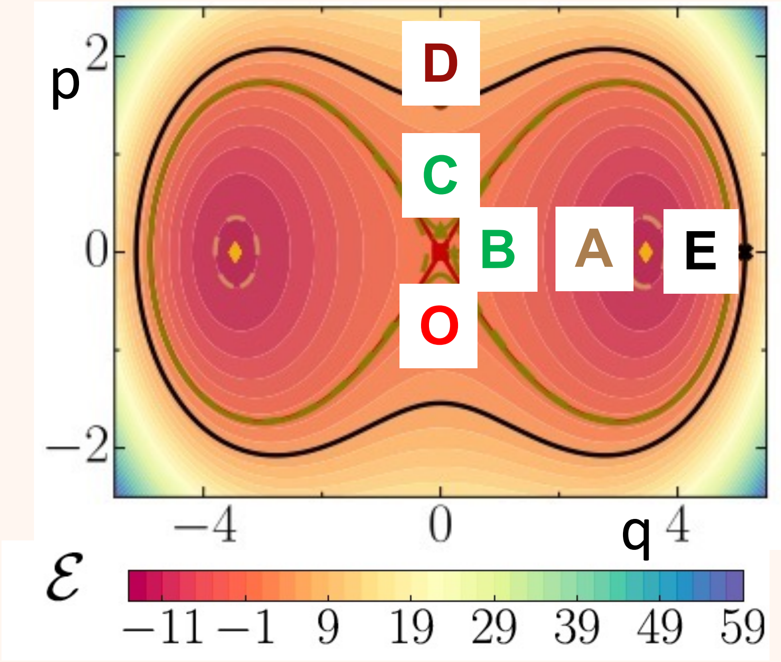


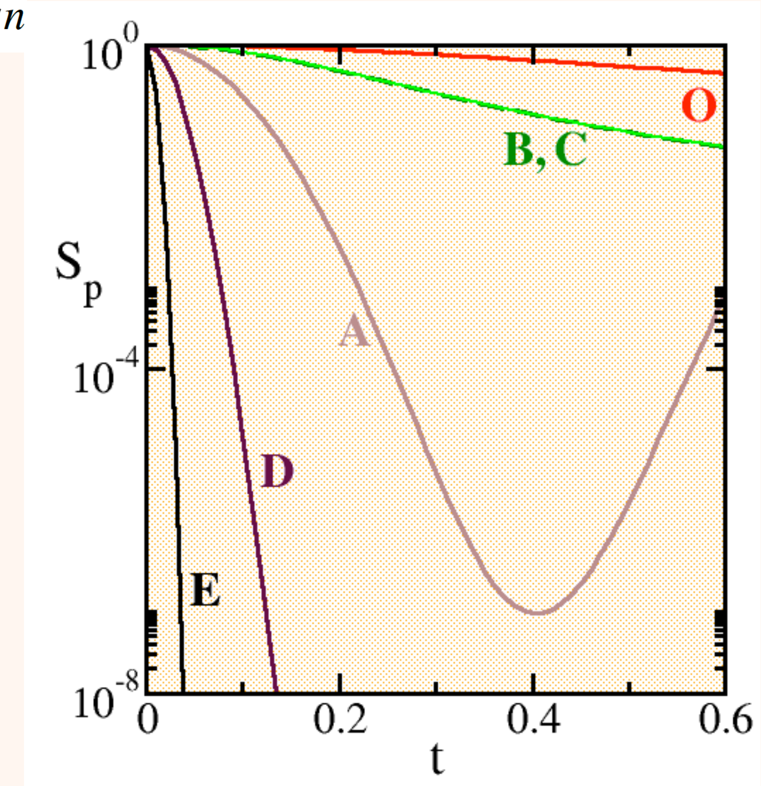


Survival Probability: D vs E



$|\langle\Psi(0)|\Psi(t)\rangle|^2$





OTOC

Out-of-time-ordered four-point correlator (**OTOC**)

$$\langle |[W(t), V(0)]|^2 \rangle \qquad \langle e^{iHt} W e^{-iHt} V e^{iHt} W e^{-iHt} V \rangle$$

OTOC

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$$\langle |[W(t), V(0)]|^2 \rangle \qquad \langle e^{iHt} W e^{-iHt} V e^{iHt} W e^{-iHt} V \rangle$$

Quantum

$$[q(t), p(0)]^2$$

Classical

$$\{q(t), p(0)\} = \partial q(t)/\partial q(0) \sim e^{\lambda_{\mathrm{cl}} t}$$

Sov. Phys. JETP **28**, 1200 (1969)

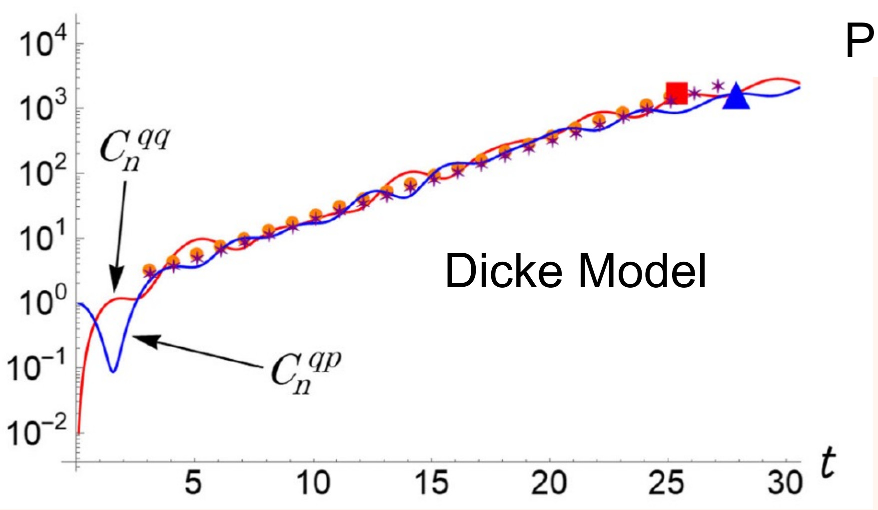
OTOC

Out-of-time-ordered four-point correlator (**OTOC**) $\langle |[W(t), V(0)]|^2 \rangle$

$$\langle e^{iHt} W e^{-iHt} V e^{iHt} W e^{-iHt} V \rangle$$

Quantum and Classical Lyapunov Exponents in Atom-Field Interaction Systems

Jorge Chávez-Carlos,[1] B. López-del-Carpio,[1] Miguel A. Bastarrachea-Magnani,[2] Pavel Stránský,[3] Sergio Lerma-Hernández,[4] Lea F. Santos,[5] and Jorge G. Hirsch[1]



PRL**122**, 024101 (2019)

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A bound on chaos JHEP (2016)

Juan Maldacena, Stephen H. Shenker, Douglas Stanford

We conjecture a sharp bound on the rate of growth of chaos in thermal quantum systems with a **large number of degrees of freedom**. Chaos can be diagnosed using an out-of-time-order correlation function closely related to the commutator of operators separated in time. We conjecture that the influence of chaos on this correlator can develop no faster than exponentially, with Lyapunov exponent

$$\lambda_L \leq 2\pi k_B T/\hbar$$

TBRE = SYK

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k\leq l, p\leq q} \langle pq|\, V \,|kl\rangle\, a_p^\dagger a_q^\dagger a_l a_k,$$

OTOC and FOTOC

Out-of-time-ordered four-point correlator (**OTOC**) $\langle |[W(t), V(0)]|^2 \rangle$

$$\langle e^{iHt} W e^{-iHt} V e^{iHt} W e^{-iHt} V \rangle$$

FOTOC

exponential growth = scrambling

$$\hat{V} = |\Psi(0)\rangle\langle\Psi(0)| \qquad \hat{W} = e^{i\delta\phi\hat{G}}$$

$\delta\phi$ small perturbation $\hat{G}$ Hermitian operator

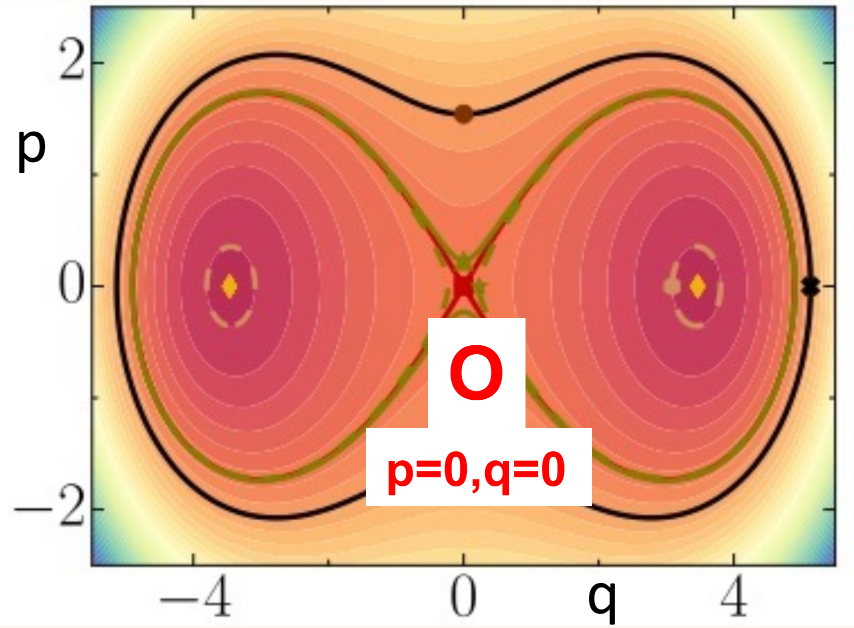
$$F_{\rm otoc}(t) = \sigma_G^2(t) = \langle \hat{G}^2(t) \rangle - \langle \hat{G}(t) \rangle^2$$

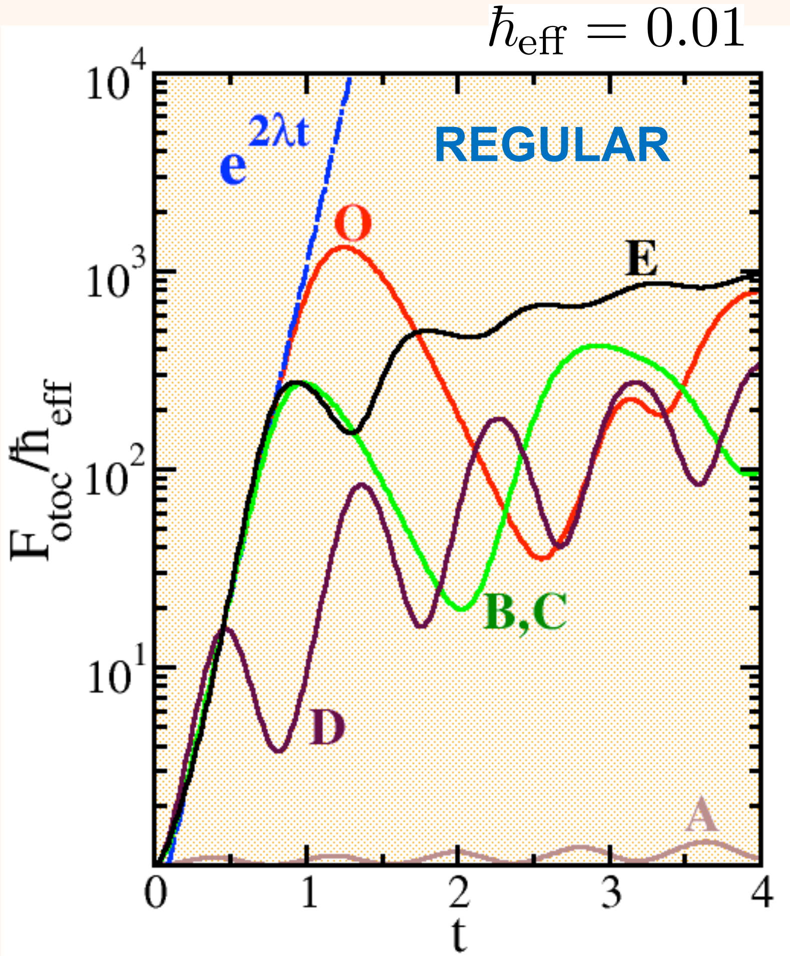
Nat. Comm. **10**, 1581 (2019).
R.J. Lewis-Swan, A. Safavi-Naini, J.J. Bollinger & A.M. Rey

FOTOC: Exponential Growth for O

Lyapunov exponent: $\lambda = 2K\xi$

$$F_{\text{otoc}}(t) = \sigma_p^2(t) + \sigma_q^2(t)$$





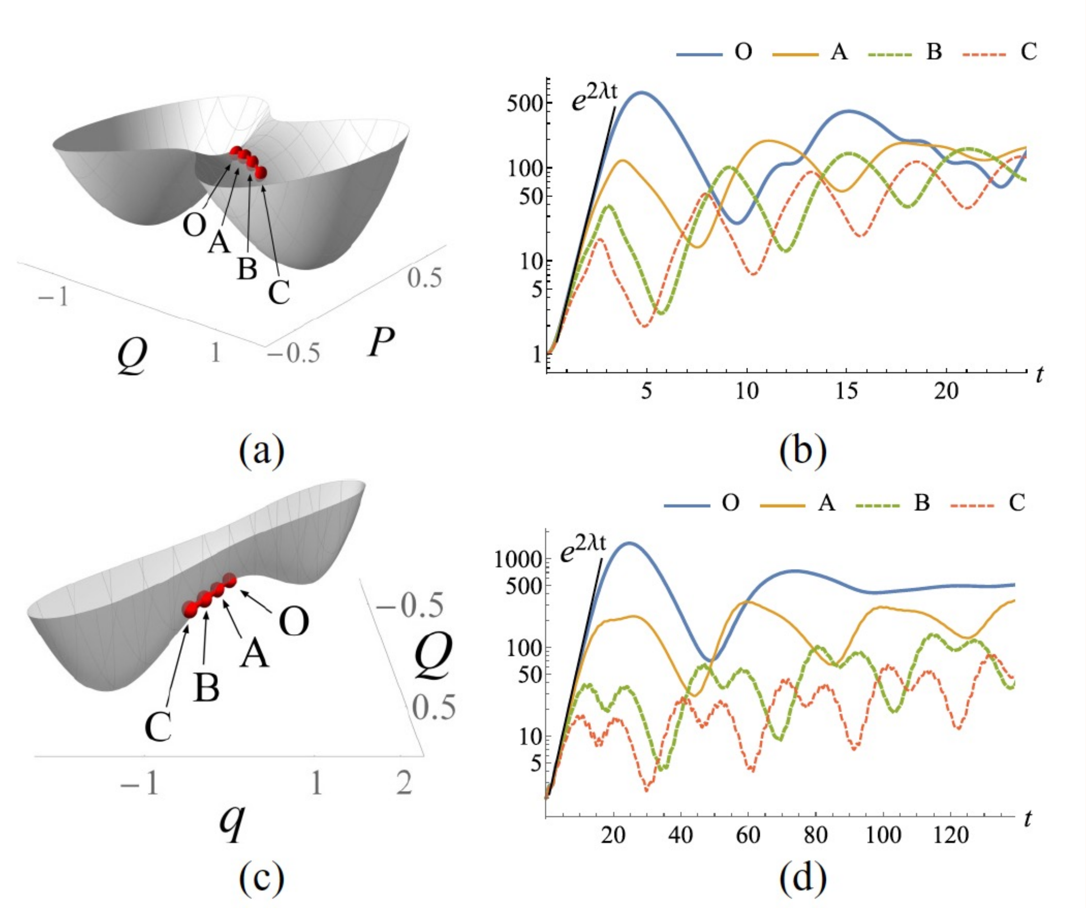
Slow Survival Probability,
Exponentially **Fast** FOTOC.

PARADOX?

Exponential Growth due to Instability

Lipkin-Meshkov-Glick model

Dicke model in the regular regime



REGULAR systems also exhibits exponential growth of OTOCs

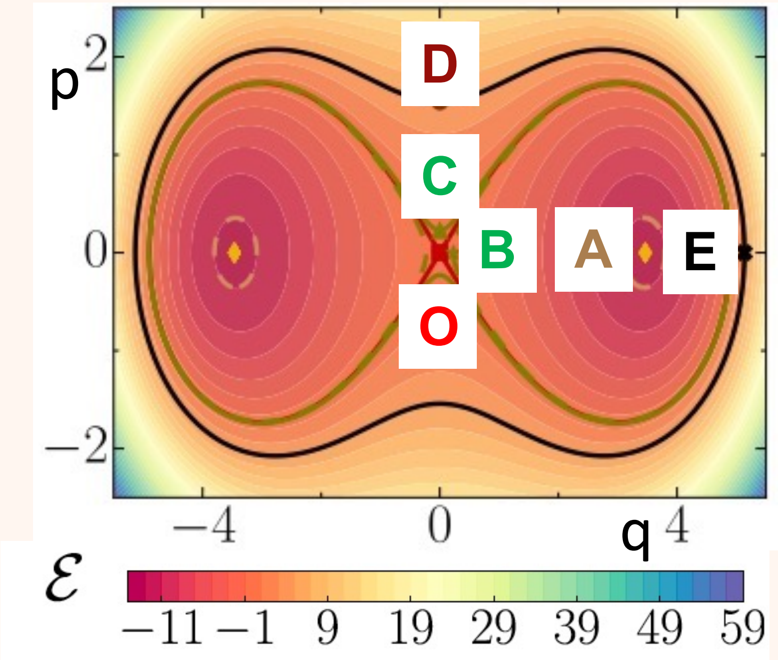
PRB 98, 134303 (2018)
PRL123,160401 (2019)
PRL124, 140602 (2020)
JHEP 2020, 68 (2020)

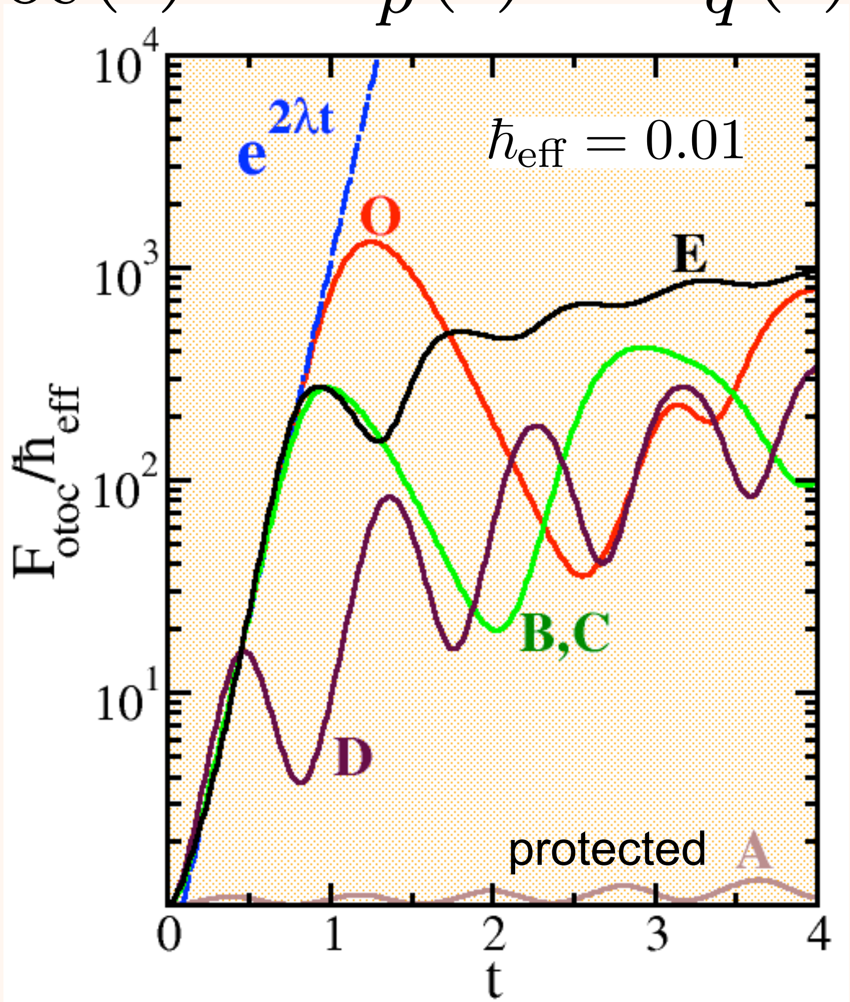
Pilatowsky et al
PRE **101**, 010202(R) (2020)

FOTOC: O vs A-E

Lyapunov exponent: $\lambda = 2K\xi$

$$F_{\rm otoc}(t) = \sigma_p^2(t) + \sigma_q^2(t)$$



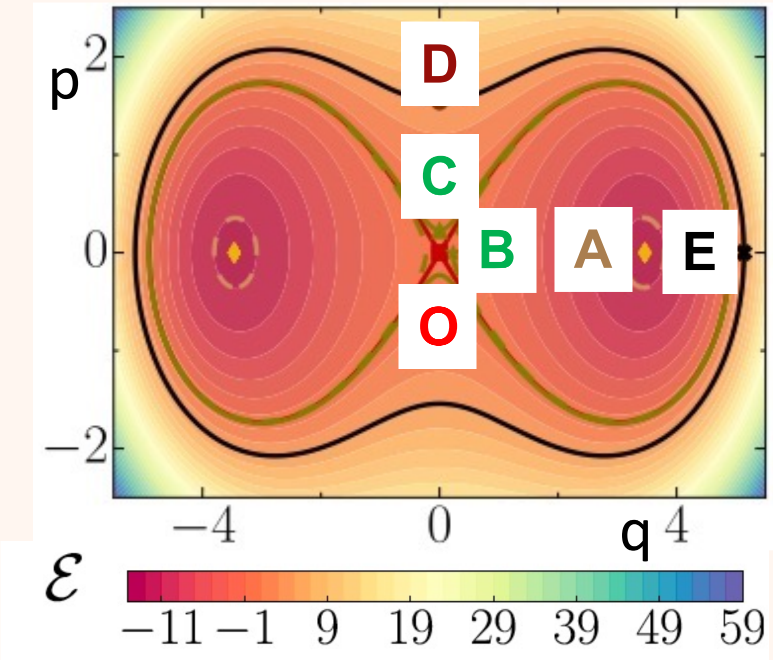


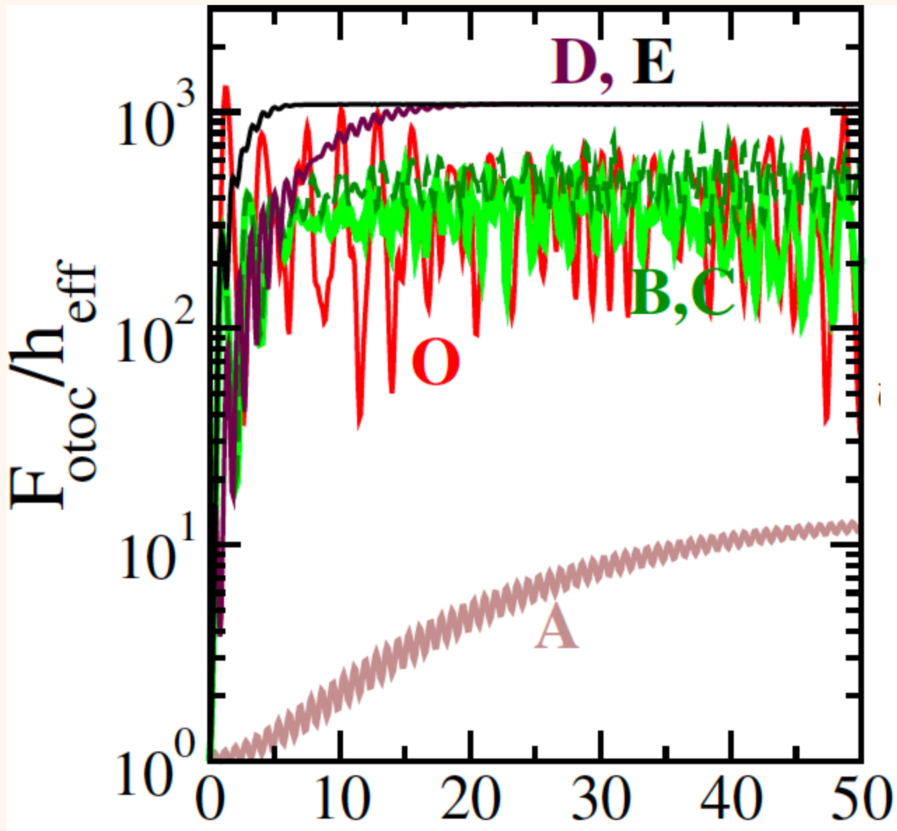
Exponential behavior of FOTOC in the **VICINITY** of the critical point.

FOTOC: O vs A-E

Lyapunov exponent: $\lambda = 2K\xi$

$$F_{\text{otoc}}(t) = \sigma_p^2(t) + \sigma_q^2(t)$$





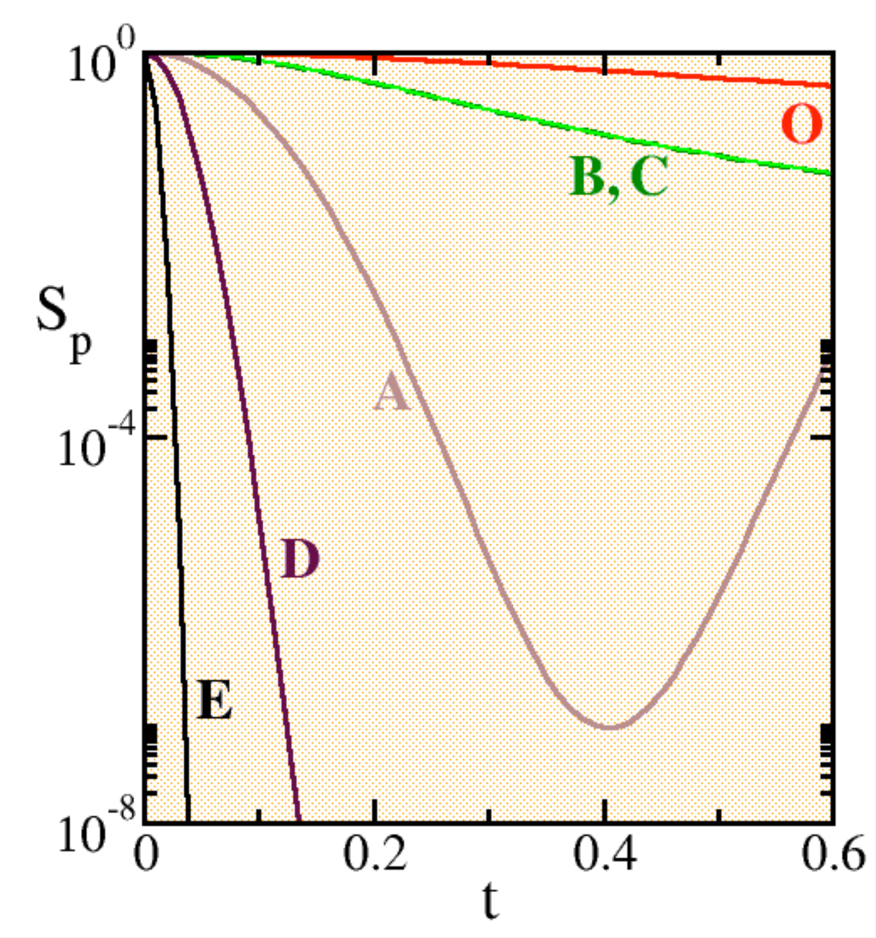
$\hbar_{\text{eff}} = 0.01$

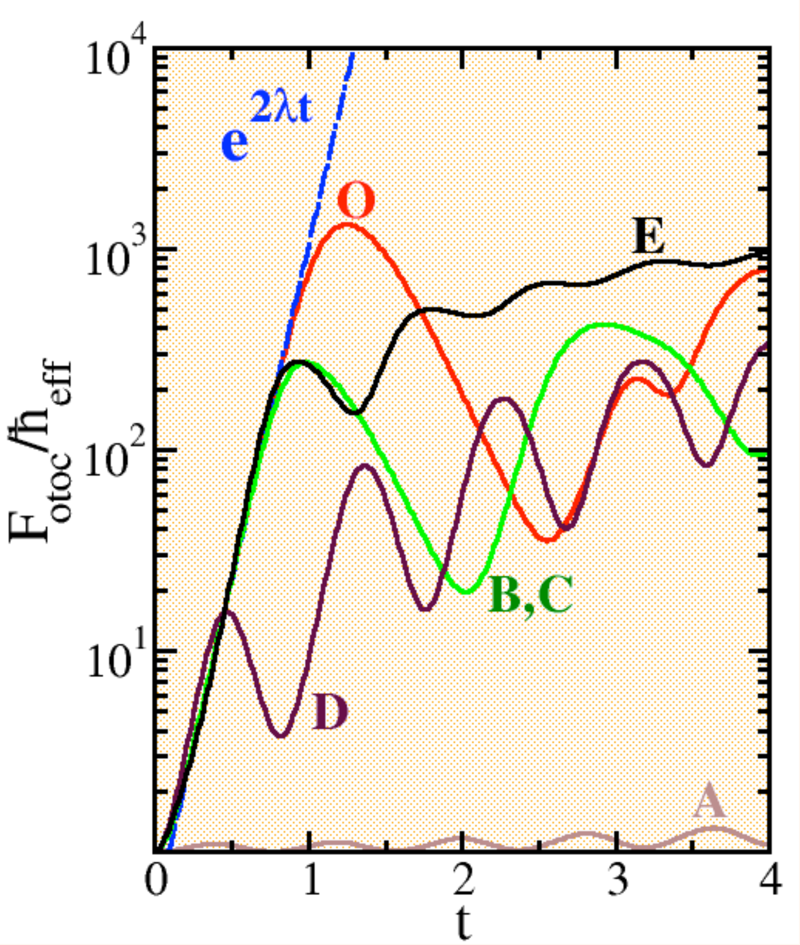
Exponential behavior = scrambling?

Paradox?

$$|\langle\Psi(0)|\Psi(t)\rangle|^2$$

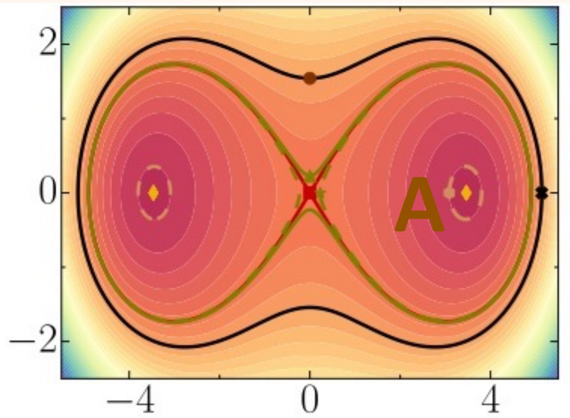
$$F_{\mathrm{otoc}}(t) = \sigma_p^2(t) + \sigma_q^2(t)$$

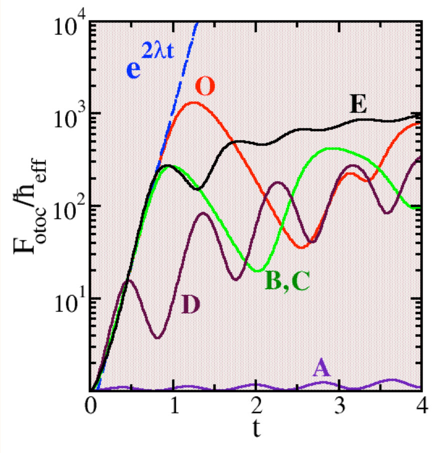


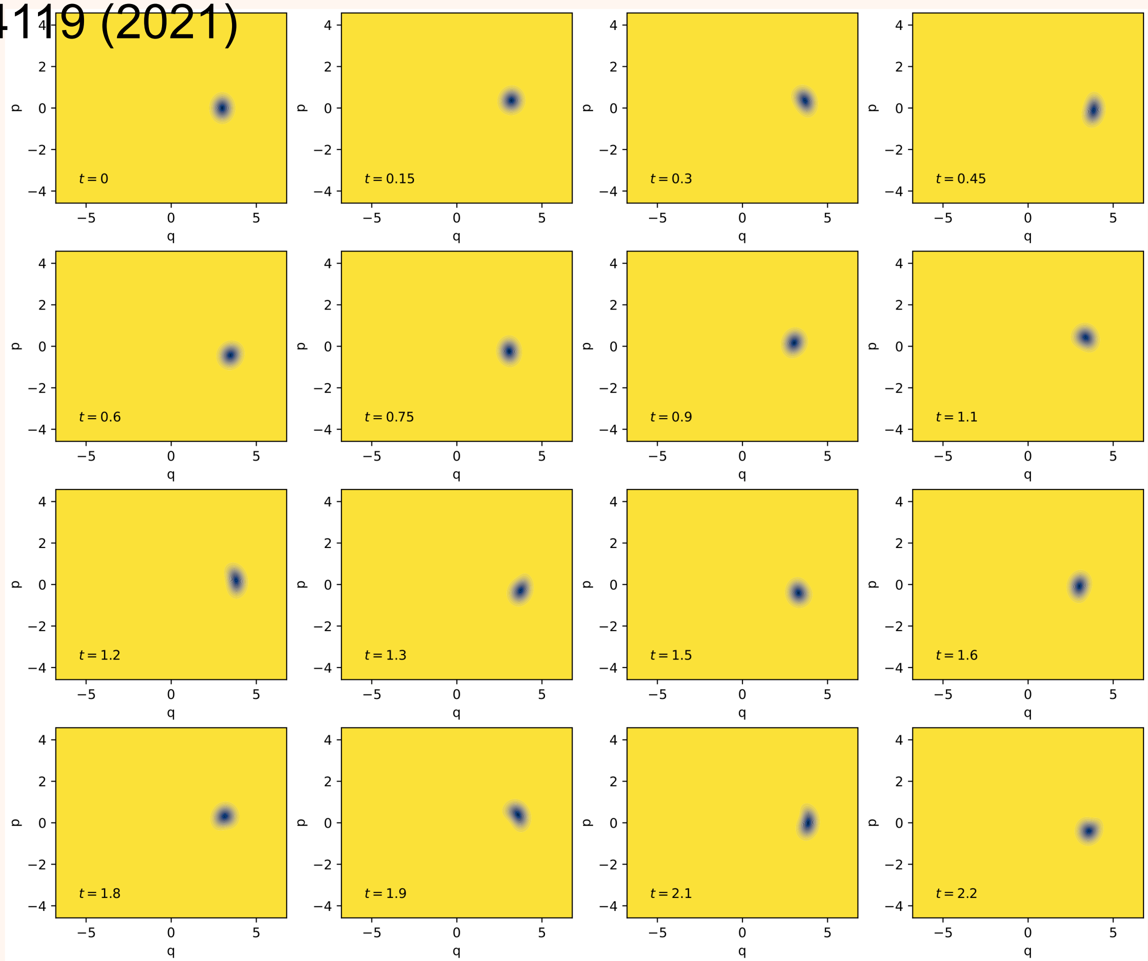


Time Evolution of the Husimi Quasidistribution

Bound Systems: PRE **104**, 034119 (2021)







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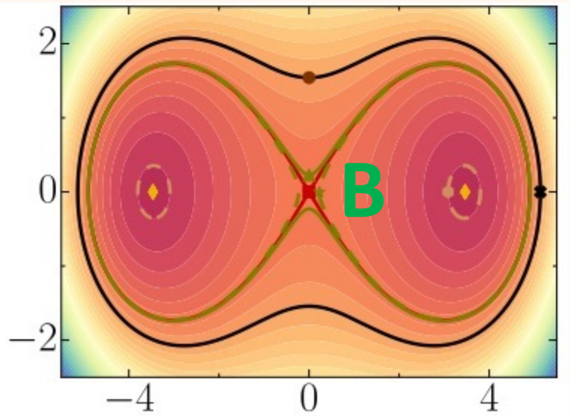
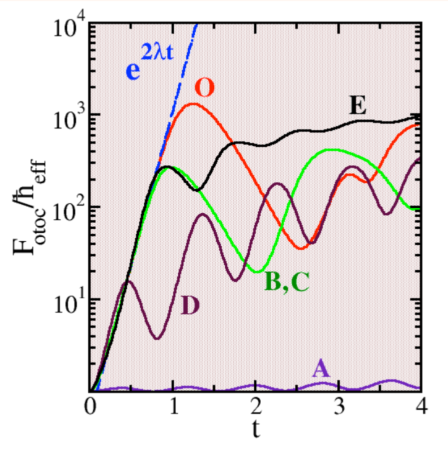
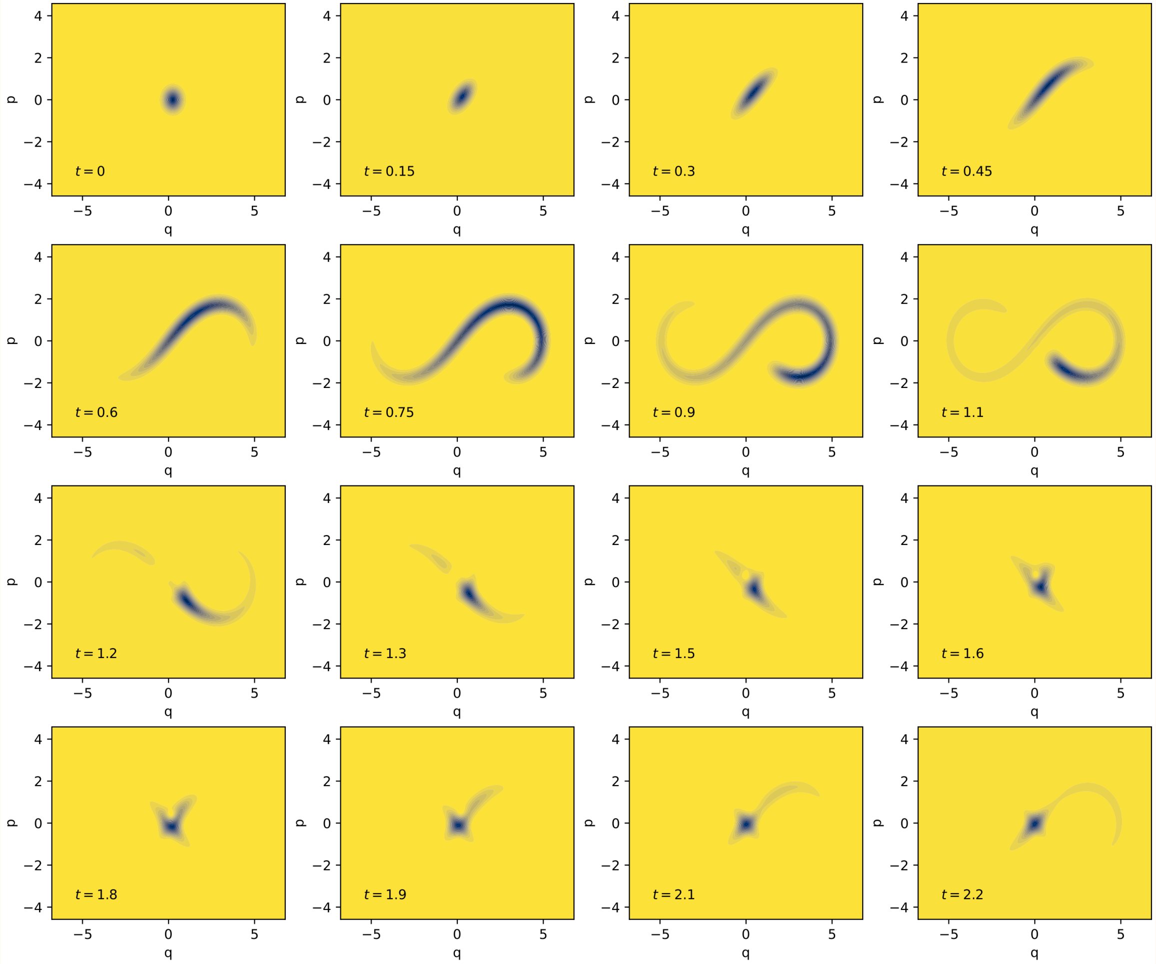
https://gitlab.com/currix1/kerr_resonator_animations

Time Evolution of the Husimi Quasidistribution

B state







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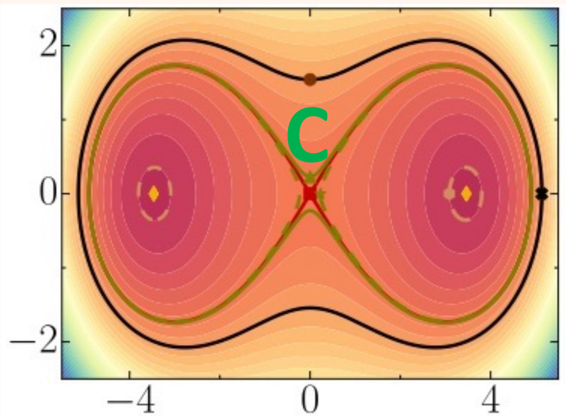
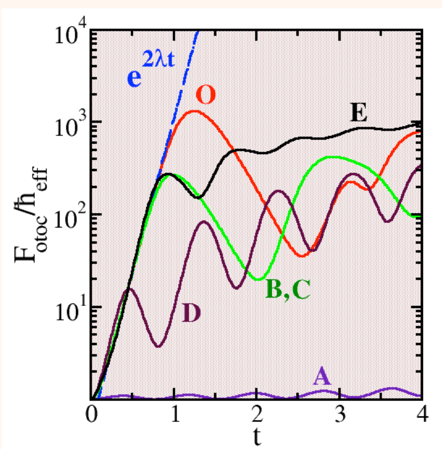
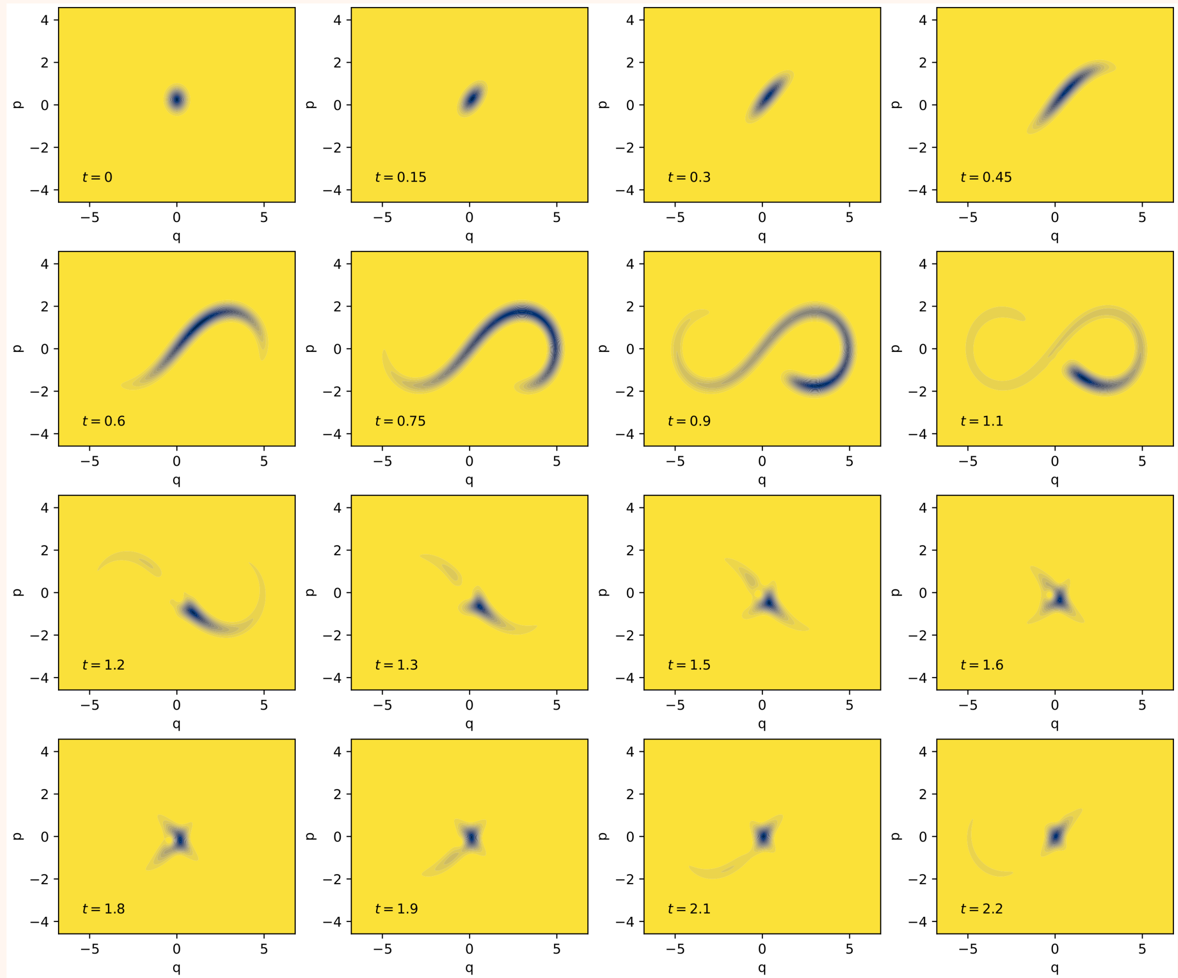
Time Evolution of the Husimi Quasidistribution

C state







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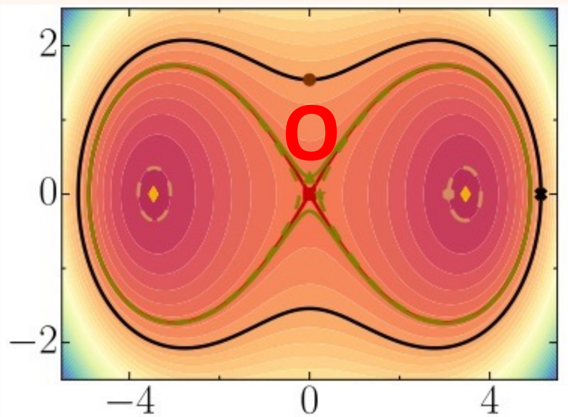
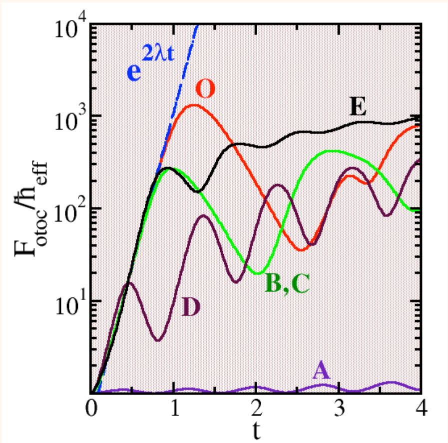
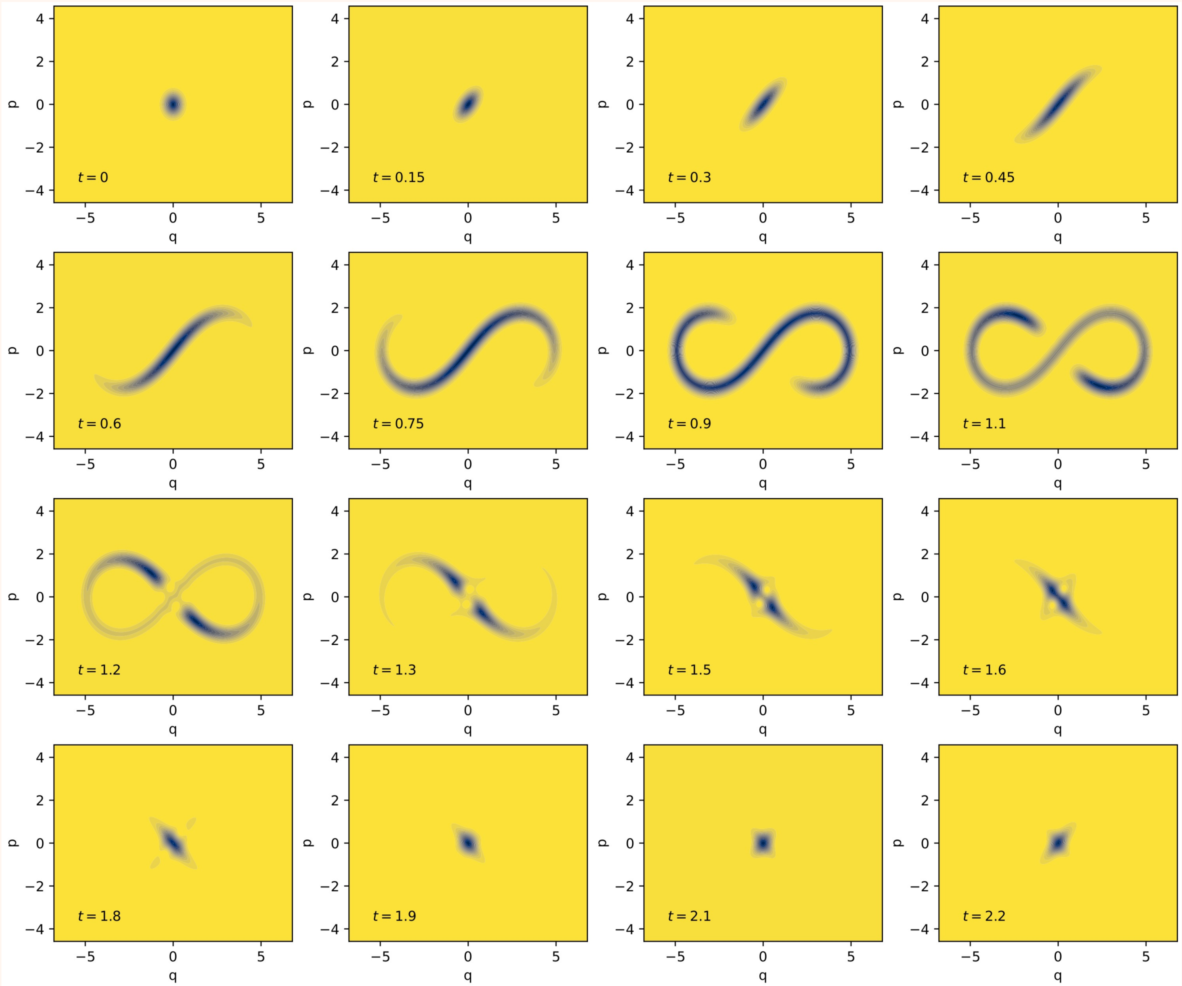
Time Evolution of the Husimi Quasidistribution

O state







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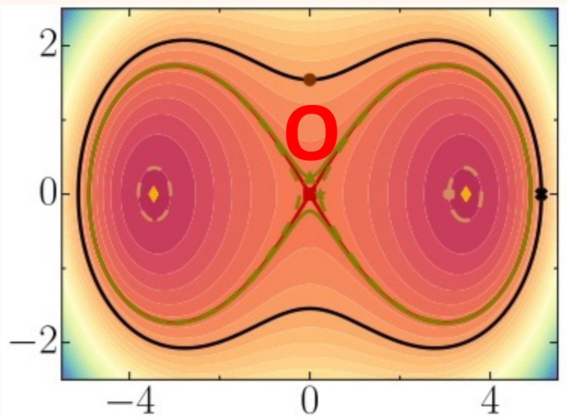
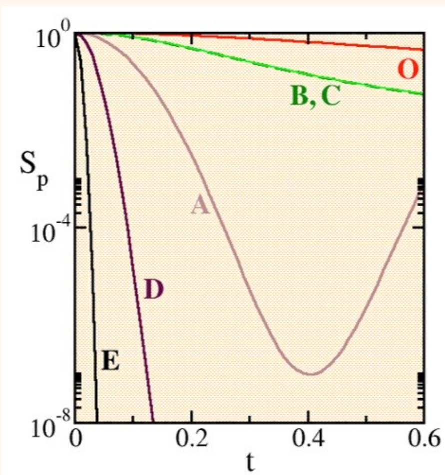
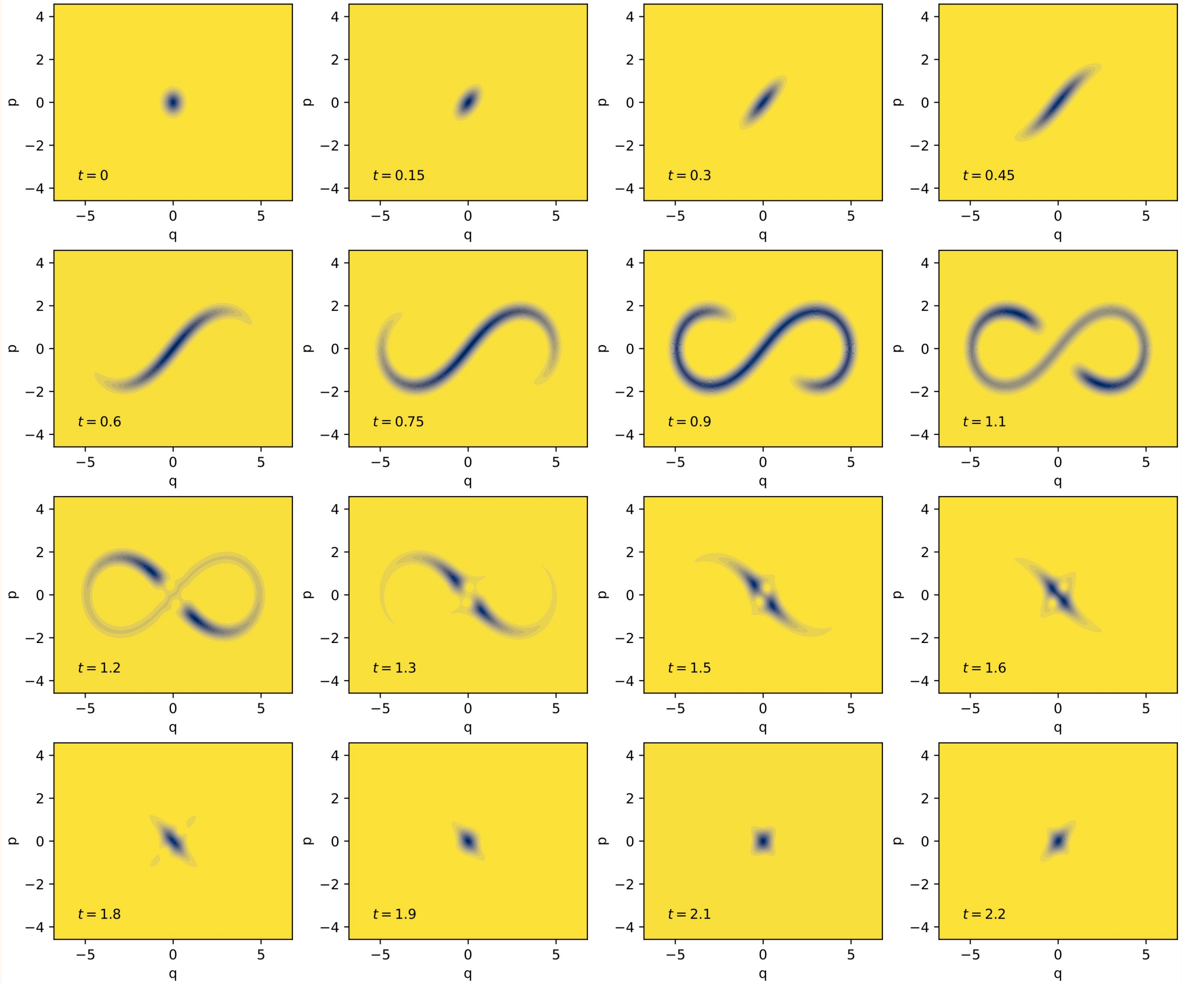
QPTN10
Dubrovnik, 2022

Time Evolution of the Husimi Quasidistribution

O state







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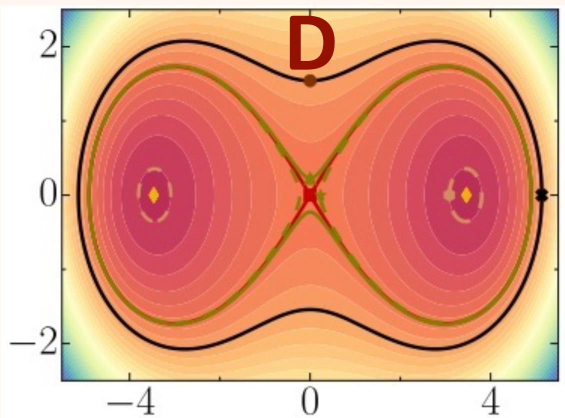
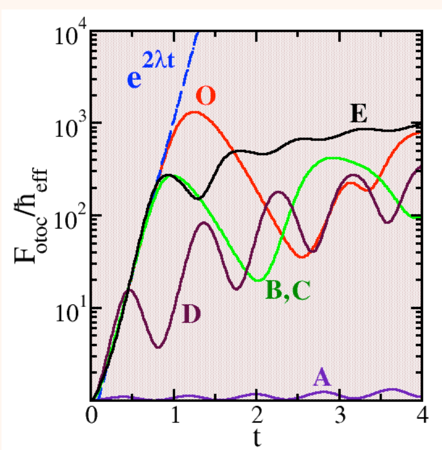
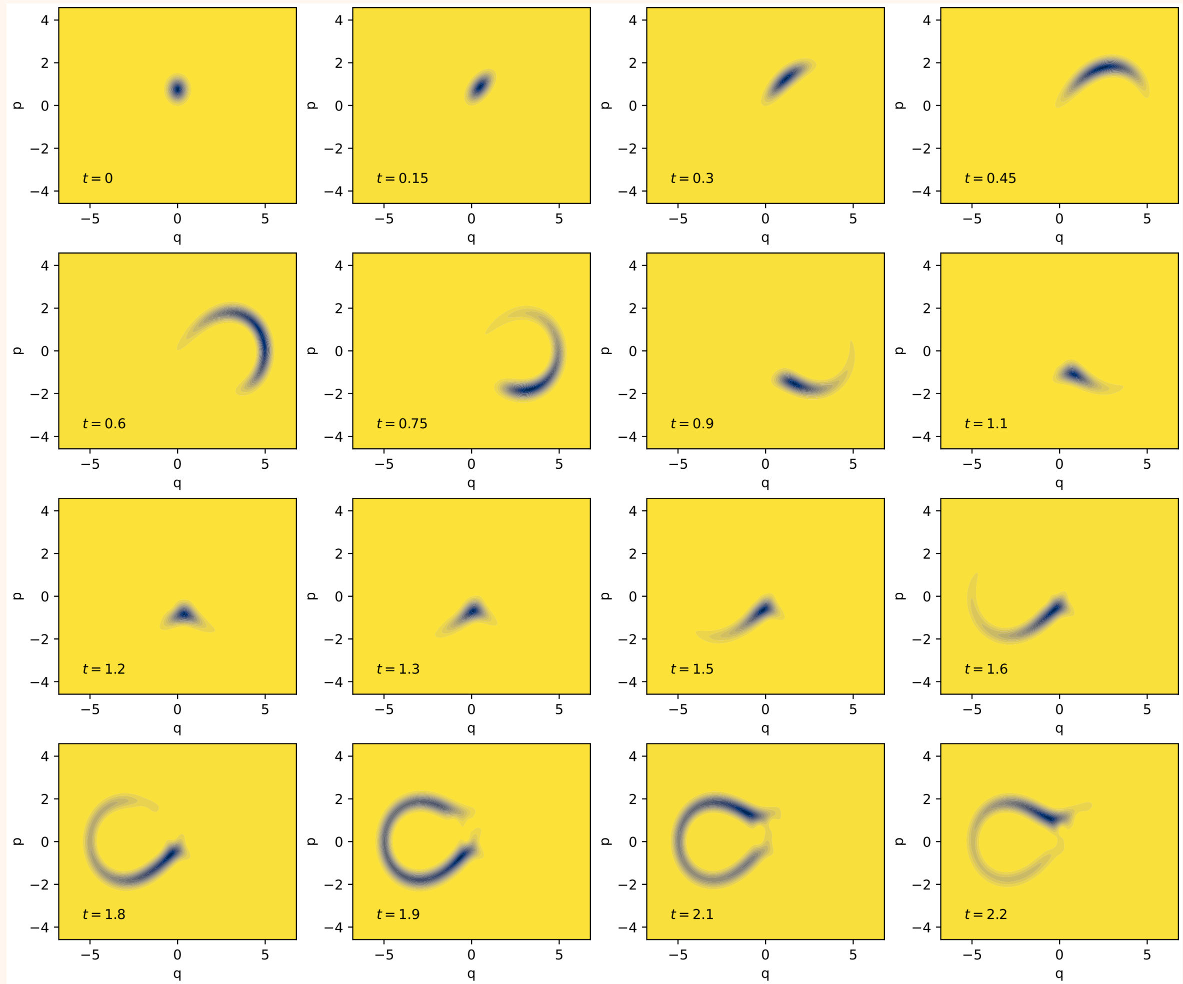
Time Evolution of the Husimi Quasidistribution

D state







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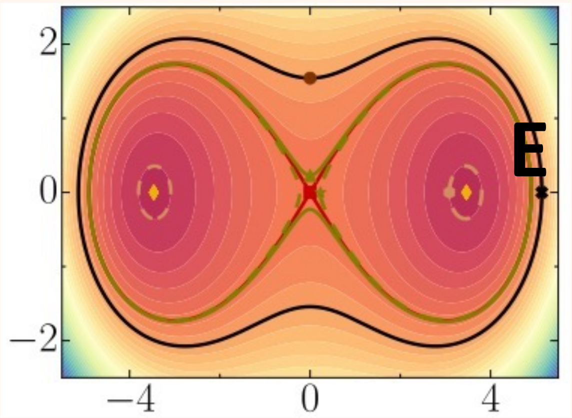
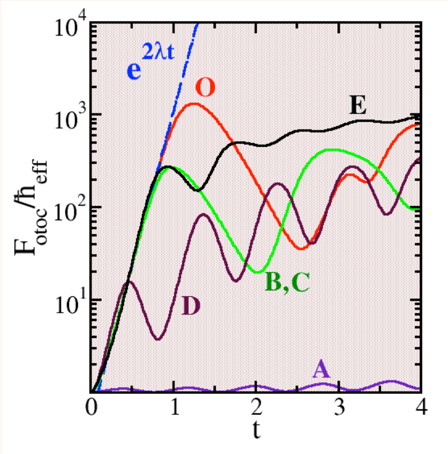
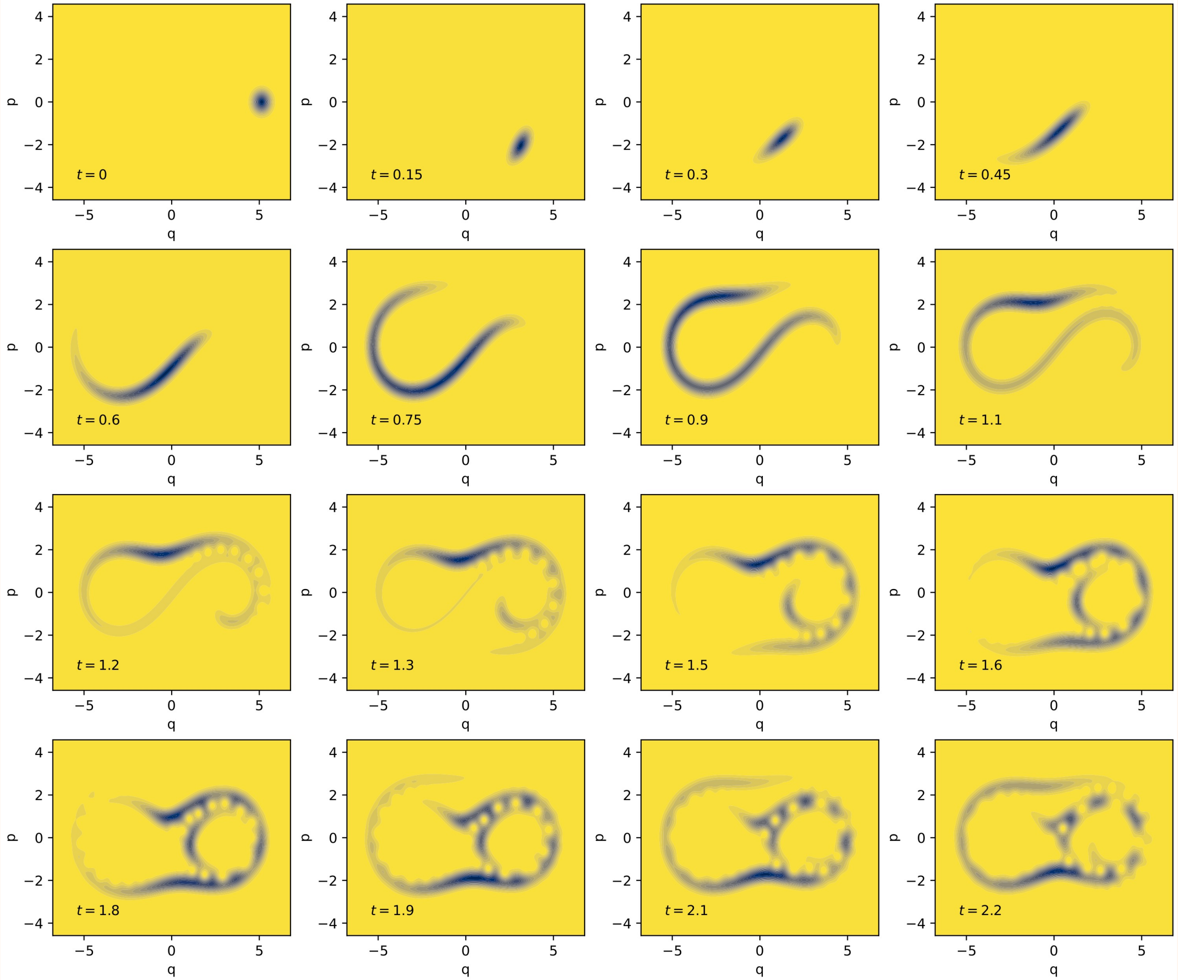
Time Evolution of the Husimi Quasidistribution

E state







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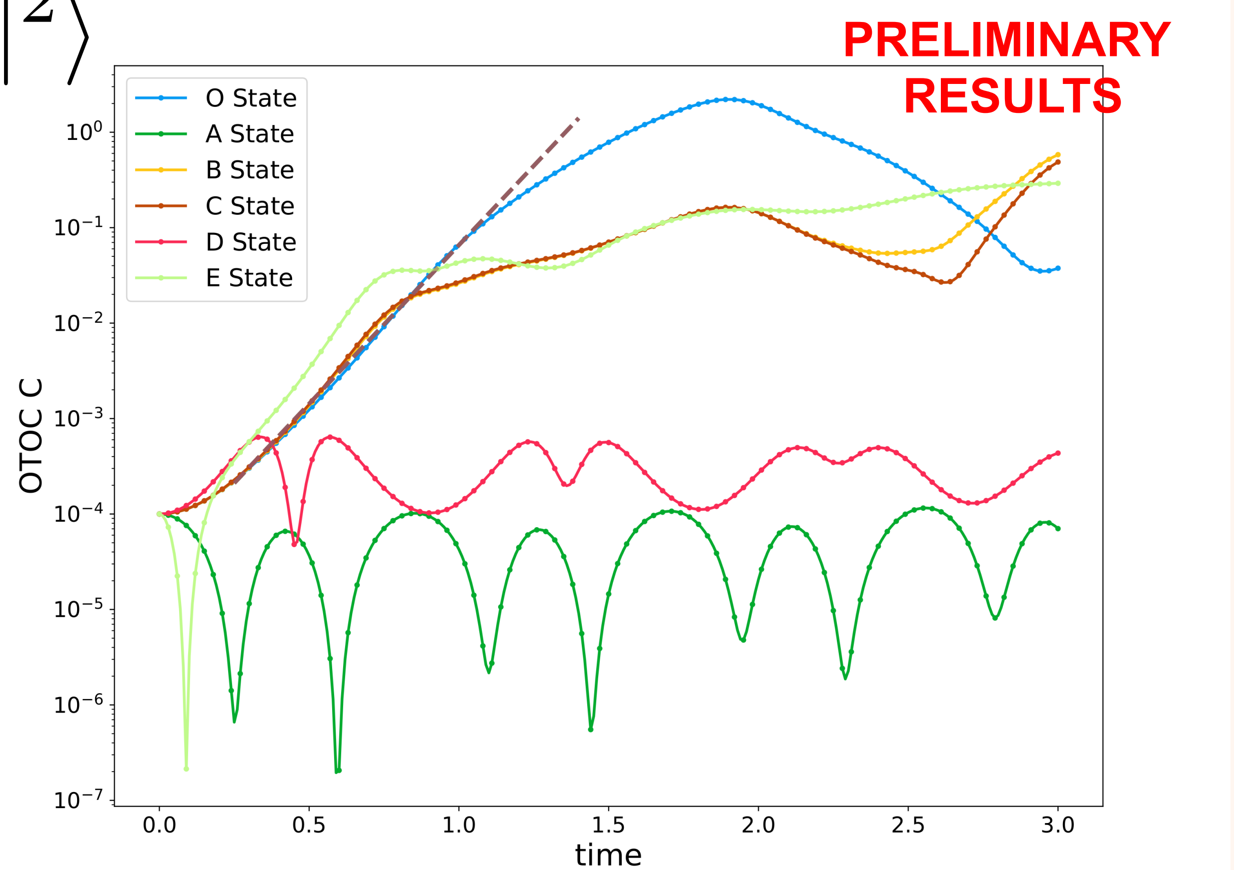
Exponential OTOC Growth

$$\langle |[W(t), V(0)]|^2 \rangle$$

$$\hat{V} = \hat{p}$$
$$\hat{W} = \hat{q}$$

$\xi = 6$

$\hbar_{\text{eff}} = 0.01$



PRE **101**, 010202(R) (2020)
JHEP **2020**, 68 (2020)

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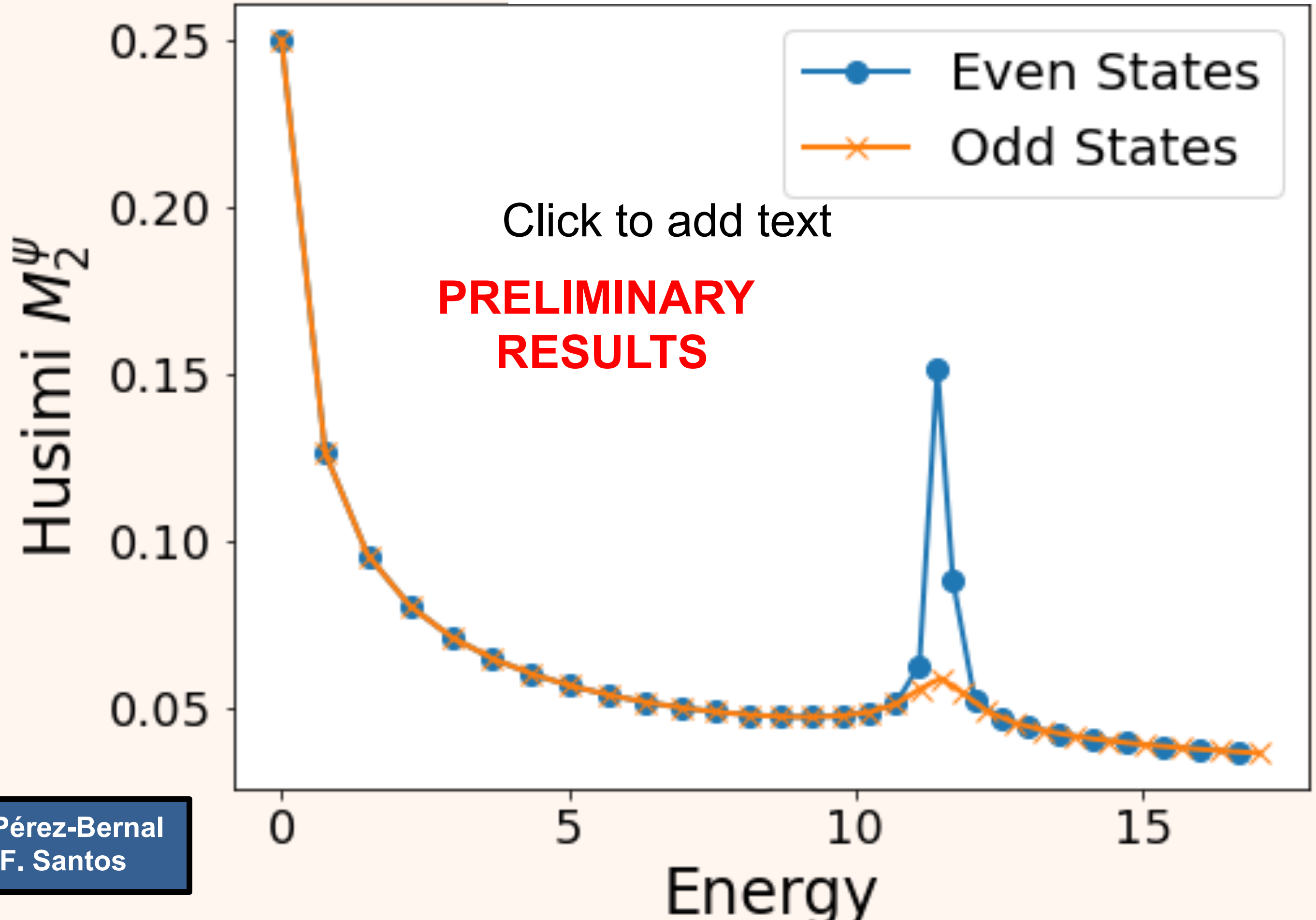
Husimi Quasidistribution Second Moment

PRE **65**, 036205 (2002)
PRE **104**, 034119 (2021)

$$M_2^{\Psi} = \frac{1}{\pi}\int d^2\alpha\, Q_{\Psi}^2(\alpha) = \frac{1}{2\hbar_{\text{eff}}\pi}\int dq\, dp\, Q_{\Psi}^2(q,p)$$



$\xi = 6$

$\hbar_{\text{eff}} = 0.1$

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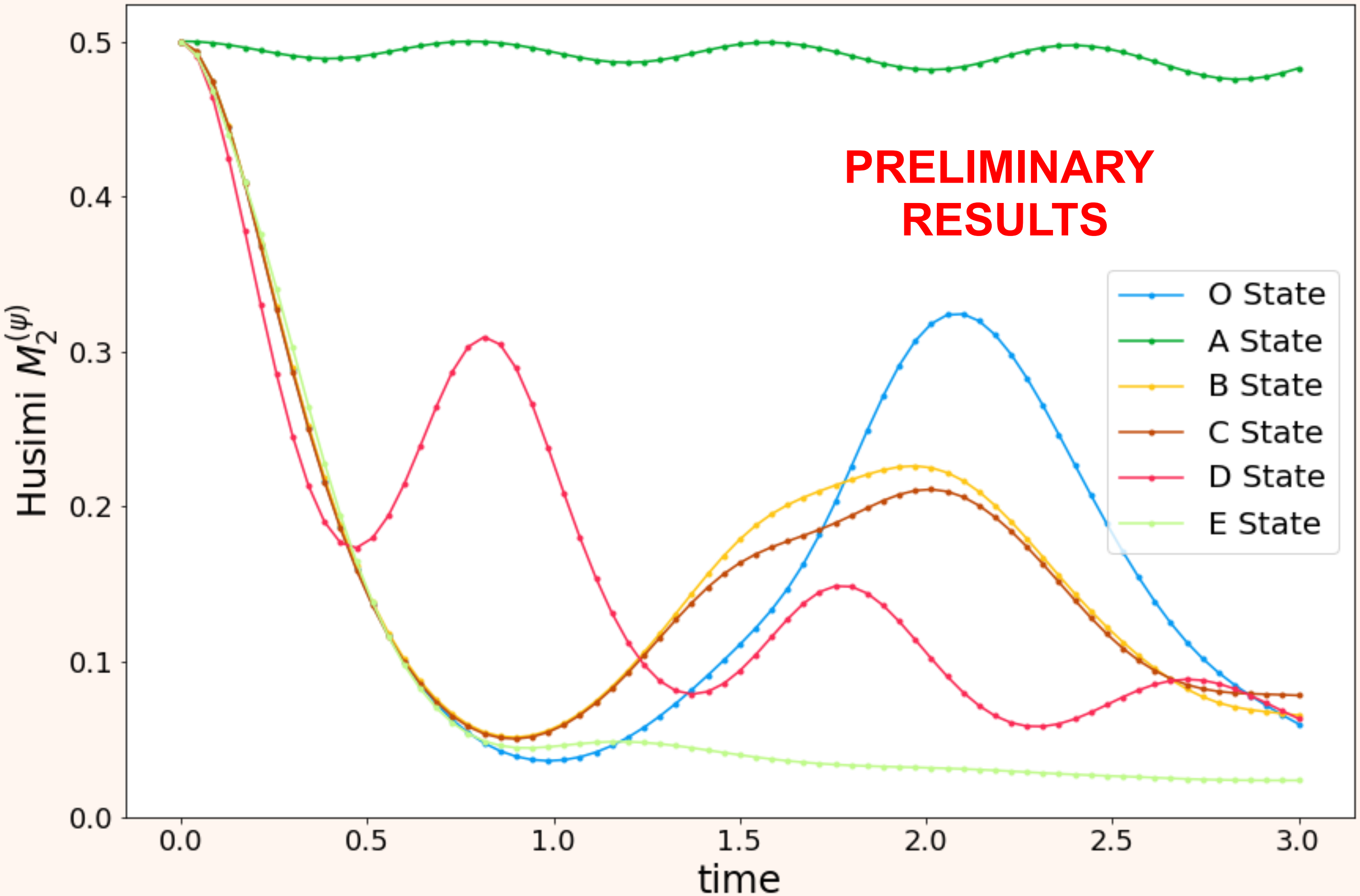
Husimi Quasidistribution Second Moment



$\xi = 6$ $\hbar_{\text{eff}} = 0.1$

Conclusions

Squeezed Kerr oscillator measures both spectrum and dynamics
exhibits signatures of ESQPT
excellent platform for ESQPT (quantum control)

The experiment for the spectrum was done.
The experiment for the dynamics will be.

Slow survival probability vs exponentially fast FOTOC.
(storage vs scrambling)

We have results for the kicked Kerr oscillator
gateway to classical and quantum chaos

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Coupling oscillators: many-body quantum dynamics

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Discussion

<u>Quantum Dynamics</u>

How long does it take for isolated many-body quantum systems perturbed far from equilibrium to finally reach equilibrium?

Quantum dynamics/equilibration/thermalization depend on many factors:

- system size
  increases/decreases with the system size
- model
  regular/chaotic, short/long-range
- initial state
  middle vs edges of the spectrum
- **QUANTITY**
  local/global, phase space/Hilbert space

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