

Proposal for many-body quantum chaos detection

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In this work, we use the term “quantum chaos” to refer to spectral correlations similar to those found in random matrix theory. Quantum chaos can be diagnosed through the analysis of level statistics using the spectral form factor, which detects both short- and long-range level correlations. The spectral form factor corresponds to the Fourier transform of the two-point spectral correlation function and exhibits a typical slope-dip-ramp-plateau structure (aka correlation hole) when the system is chaotic. We discuss how this structure could be detected through the dynamics of two physical quantities accessible to experimental many-body quantum systems: the survival probability and the spin autocorrelation function. When the system is small, the dip reaches values that are large enough at times which are short enough to be detected with current experimental platforms and commercially available quantum computers.

I. INTRODUCTION

The main mechanism for the onset of quantum chaos in many-body quantum systems is the interactions between particles. Similar to what one finds in random matrix theory, realistic many-body quantum systems in the chaotic regime are characterized by correlated energy levels [1] and middle-spectrum eigenstates that approach random vectors by filling the energy shell [2]. When these systems are taken far from equilibrium, quantum chaos underlies the spread and scrambling of quantum information, hindering the reconstruction of the initial state through local measurements. Such redistribution of quantum information is intertwined with the thermalization of subsystems [2, 3] and the difficulty in reaching a localized phase [4, 5]. Understanding and quantifying many-body quantum chaos is thus essential for describing and controlling many-body quantum dynamics and for the development of quantum technologies. In this work, we discuss how spectral correlations indicative of quantum chaos can be experimentally detected via the dynamics of many-body quantum systems.

The spectral form factor provides direct access to short- and long-range correlations among the eigenvalues. Mathematically, it is defined as the Fourier transform of the two-point function of the energy spectrum [6]. In chaotic systems, it presents a slope-dip-ramp-plateau structure analogous to the one found in random matrix theory, therefore signaling a rigid spectrum. This structure can persist even in the presence of environmental noise [7–13]. The ramp only appears when the levels are correlated and the plateau represents the saturation value of the spectral form factor. The ramp reaches the plateau at the Heisenberg time, which is inversely proportional

to the mean level spacing and thus proportional to the dimension of the Hilbert space.

The analysis of level statistics through the spectral form factor is an excellent way to detect many-body quantum chaos in experiments with access to the spectrum, as in nuclear physics. Level statistics is a less efficient diagnostic tool of chaos in experiments with cold atoms [14–19], ion traps [20–23] and available quantum computers [24, 25], where the spectrum is not easily accessible and the focus is instead on many-body quantum dynamics. To detect the slope-dip-ramp-structure through dynamics, recent works have proposed to monitor the fidelity of thermofield double states evolved under the Sachdev-Ye-Kitaev model [8] and to use measurement protocols on evolved random product states [26, 27] that are amenable to spin models realizable in platforms of Rydberg atoms [28], superconducting qubits [29] and stroboscopically-driven cold atoms in optical lattices [30].

Our approach to detect many-body quantum chaos is suitable to different experiments that study dynamics. We probe two dynamical quantities that can be experimentally measured and that, like the spectral form factor, exhibit the characteristic slope-dip-ramp-plateau structure when the system is chaotic. They are the survival probability and the spin autocorrelation function.

The survival probability is defined as the squared absolute value of the overlap between the initial state and its evolved counterpart. The spectral form factor can be interpreted as the survival probability of an initial thermofield double state or a coherent Gibbs state [8, 31, 32], but the preparation of such initial states is not straightforward in current experiments. Instead, we investigate the survival probability of experimentally accessible initial states.

The idea of detecting quantum chaos through the survival probability was first proposed in Ref. [33], where the slope-dip-ramp-plateau structure was originally known as the “correlation hole” [8, 10, 33–51]. It was later shown that the correlation hole emerges also in the spin autocorrelation function [43, 44], which contrary to the survival probability, is local in real space.

We study the evolution of the survival probability and the spin autocorrelation function in two different many-body spin-1/2 models that can be realized in current experiments with cold atoms, ion traps, nuclear magnetic resonance (NMR) platforms [52, 53], and in digital quantum computers. They correspond to the one-dimensional (1D) disordered spin-1/2 Heisenberg model and the 1D disordered long-range Ising model in a transverse field. When the disorder strength is comparable to the interaction strength, these systems are chaotic.

The main challenges of our proposal lie on the minimum value of the correlation hole and the timescale for its appearance. The lowest value (timescale) decreases (increases) with the dimension of the Hilbert space, which in turn grows exponentially with the size of our many-body quantum systems. Nevertheless, we show that the correlation hole can emerge even when our systems have only 6 or 8 sites. For such small chains, the dip happens at sufficiently large values and the Heisenberg time is sufficiently short for the potential detection of many-body quantum chaos with current experimental capabilities. Our analysis includes the effects of shot noise.

Due to the small Heisenberg time of small chains, the quantum circuits for the time evolution of our spin-1/2 models should be relatively shallow, allowing for the implementation in current commercial quantum computers. We demonstrate this possibility through noiseless time evolution of the Heisenberg model with 6 qubits in Qiskit.

In addition to the many-body spin models, we also present results for a disordered spin-1/2 chain with a single excitation and nearest-neighbor coupling. This system is analogous to the one-particle Anderson model, thus being localized in the thermodynamic limit for an infinitesimal disorder. However, when the chain is finite, it can present level correlations that get manifested in the dynamics and could be experimentally detected.

The paper is organized as follows. In Sec. II, we review the definition and properties of the spectral form factor, survival probability, and spin autocorrelation function. In Sec. III, we analyze the dynamics of the survival probability and the spin autocorrelation function for the 1D disordered isotropic Heisenberg spin-1/2 model with nearest-neighbor couplings and the 1D disordered long-range Ising model in a transverse field. We also compare the numerical results for the survival probability evolving under the Heisenberg model with results from Qiskit, and discuss how the distributions of measurements of the survival probability at a few times might suffice for the detection of the correlation hole. In Sec. IV, the analysis is extended to the 1D spin-1/2 model with a single

excitation. Conclusions are presented in Sec. V.

II. DYNAMICAL INDICATORS OF MANY-BODY QUANTUM CHAOS

The two-point spectral form factor captures both short- and long-range correlations in the energy spectrum, thus providing a complete diagnostic of quantum chaos. This quantity has also been used to question the existence of a many-body localized phase [5] and in recent studies of scale-invariant critical dynamics [54]. The two-point spectral form factor is defined as [6],

$$\text{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{m,n} e^{i(E_m - E_n)t} \right\rangle, \quad (1)$$

where $\hbar = 1$, D is the dimension of the Hamiltonian matrix that describes the system, E_n represents its eigenvalues, and $\langle \cdot \rangle$ indicates an ensemble average. For large random matrices from the Gaussian orthogonal ensemble (GOE), we have [43, 44]

$$\text{SFF}(t) \simeq \frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - \frac{1}{D} b_2 \left(\frac{\Gamma t}{2D} \right) + \frac{1}{D}, \quad (2)$$

where $\mathcal{J}_1(t)$ is the Bessel function of the first kind, Γ is the width of the semicircular density of states, and $b_2(t)$ is the two-level form factor [6],

$$b_2(t) = \begin{cases} 1 - 2t + t \log(1 + 2t), & t \leq 1, \\ t \log \left(\frac{2t + 1}{2t - 1} \right) - 1, & t > 1. \end{cases} \quad (3)$$

In Fig. 1, we show the spectral form factor averaged over an ensemble of GOE random matrices. The first term in Eq. (2) represents the slope in the slope-dip-ramp-plateau structure. The slope exhibits oscillations characteristic of the Bessel function, whose envelope decays as a powerlaw $\propto t^{-3}$, as seen in Fig. 1. The dip corresponds to the minimum value of $\text{SFF}(t)$, and the ramp is the region that follows the dip, being below the saturation value (plateau) at $1/D$. The interval where $\text{SFF}(t)$ is below the plateau corresponds to the correlation hole.

The ramp is described by the two-level form factor in Eq. (3) and it emerges due to spectral correlations. Unless averages are performed, the dip-ramp structure is hidden by fluctuations, because the spectral form factor is non-self-averaging [12, 55–58]. The beginning of the ramp and its end at the Heisenberg time are marked with vertical lines in Fig. 1.

It is usual to add a filter to the spectral form factor, a common choice being the Boltzmann factors [32],

$$f(E_n) = \frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}}, \quad (4)$$

where β is the inverse temperature. The case in Eq. (1) is recovered for infinite temperature, $\beta = 0$.

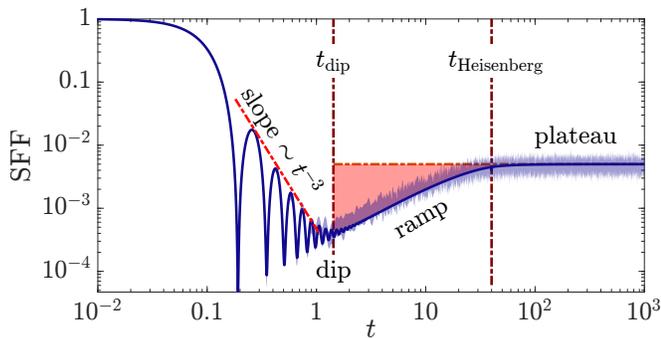


FIG. 1. Slope-dip-ramp-plateau structure (correlation hole) of the spectral form factor obtained with an ensemble of 100 GOE random matrices of dimension $D = 200$. The elements of the matrices have $\langle H_{ij} \rangle = 0$, $\langle H_{ij}^2 \rangle = 1/2$ for $i \neq j$, and $\langle H_{ii}^2 \rangle = 1$, so that $\Gamma \sim \sqrt{D/2}$. The dark solid line is the analytical expression in Eq. (2) and the shaded curve is the average over the ensemble. Vertical dashed lines mark the beginning and the end of the ramp.

A. Survival probability

The survival probability is defined as

$$S_P(t) = \left| \langle \Psi(0) | \Psi(t) \rangle \right|^2 = \sum_{m,n} |c_m|^2 |c_n|^2 e^{i(E_m - E_n)t}, \quad (5)$$

where $c_n = \langle E_n | \Psi(0) \rangle$ is the n th component of the initial state $|\Psi(0)\rangle$ written in the energy eigenbasis $\{|E_n\rangle\}$ of the Hamiltonian \hat{H} that describes the system. If one equates the product of components $|c_m|^2 |c_n|^2$ with the product of Boltzmann factors, $f(E_m)f(E_n)$, in Eq. (4), then the spectral form factor can be interpreted as the survival probability of an initial Gibbs state [32]. However, this is not an easy state to prepare experimentally, so we focus instead on experimentally accessible initial states evolving under physical many-body quantum systems, as specified in Sec. III.

We study quench dynamics, where the initial state is an eigenstate of the unperturbed Hamiltonian \hat{H}_0 and it evolves according to the total Hamiltonian,

$$\hat{H} = \hat{H}_0 + \lambda \hat{V}, \quad (6)$$

where \hat{V} is the perturbation and λ is the strength of the perturbation. In many-body quantum systems with two-body couplings, as considered in this work, the density of states of \hat{H} is Gaussian [59].

When the system is perturbed far from equilibrium ($\lambda \sim 1$), the energy distribution of the initial state, which is often referred to as local density of states (LDOS),

$$\rho_0(E) = \sum_{n=1}^D |c_n|^2 \delta(E - E_n), \quad (7)$$

is also Gaussian [60, 61]. In the equation above, D is the dimension of the Hilbert space. The width Γ of the

LDOS is obtained as [62]

$$\begin{aligned} \Gamma^2 &= \langle \Psi(0) | \hat{H}^2 | \Psi(0) \rangle - \langle \Psi(0) | \hat{H} | \Psi(0) \rangle^2 \quad (8) \\ &= \sum_{n \neq n_0}^D \left| \langle \varepsilon_n | \hat{H} | \Psi(0) \rangle \right|^2, \end{aligned}$$

where $|\varepsilon_n\rangle$ are the eigenstates of \hat{H}_0 and n_0 corresponds to the index of the initial state, $|\Psi(0)\rangle = |\varepsilon_{n_0}\rangle$. Notice that the calculation of Γ only requires knowledge of the off-diagonal elements, $\langle \varepsilon_n | \hat{H} | \Psi(0) \rangle$, of the total Hamiltonian \hat{H} written in the basis of eigenstates of \hat{H}_0 .

The survival probability in Eq. (5) can be equivalently expressed in terms of the Fourier transform of the LDOS,

$$S_P(t) = \left| \int_{E_{\min}}^{E_{\max}} \rho(E) e^{-iEt} dE \right|^2, \quad (9)$$

where E_{\min} and E_{\max} are the lower and upper energy bounds of the LDOS. In terms of Eq. (9), it becomes clear that the initial decay of the survival probability is Gaussian, $e^{-\Gamma^2 t^2}$, followed by a power-law decay that presents oscillations [31, 63, 64]. This is the slope of the slope-dip-ramp-plateau structure.

Since $S_P(t)$ is non-self-averaging [56, 57], we work with the averaged survival probability, $\langle S_P(t) \rangle$ to capture the subsequent features of the slope-dip-ramp-plateau structure. In strongly chaotic many-body quantum systems with time-reversal symmetry, $\langle S_P(t) \rangle$ reaches a minimum value at a time $t_{\text{dip}} \propto D^{2/3}/\Gamma$ [44], after which a ramp emerges. The ramp is closely described by the $b_2(t)$ function in Eq. (3), and it persists up to the Heisenberg time, $t_H \propto D/\Gamma$ [44]. This is the largest timescale of the system. Beyond t_H , the averaged survival probability exhibits small fluctuations around its infinite-time average, $\overline{S_P} = \langle \sum_{n=1}^D |c_n|^4 \rangle$.

B. Spin autocorrelation function

The spin autocorrelation function also detects the correlation hole [43, 44] and is also non-self-averaging at long times [56–58]. This quantity is defined as

$$I_z(t) = \frac{1}{L} \sum_{k=1}^L \langle \Psi(0) | \hat{\sigma}_k^z e^{i\hat{H}t} \hat{\sigma}_k^z e^{-i\hat{H}t} | \Psi(0) \rangle, \quad (10)$$

where L is the number of sites of the spin-1/2 chains that we consider and $\hat{\sigma}_k^z$ is the Pauli operator acting on site k . We denote the asymptotic value of the average $\langle I_z(t) \rangle$ as $\overline{I_z}$.

Like the survival probability, the spin autocorrelation function is nonlocal in time, but contrary to survival probability, it is local in real space. Another difference between the two quantities is that the correlation hole fades away for the spin autocorrelation function as

the system size increases [47]. In NMR platforms, this quantity can be directly measured for initially mixed states [53]. Upon choosing a Néel state as the initial state, $I_z(t)$ is comparable to the density imbalance experimentally probed in cold atoms [16, 18].

For both quantities, $\langle S_P(t) \rangle$ and $\langle I_z(t) \rangle$, the dynamics need to resolve the discreteness of the spectrum for the emergence of the correlation hole. This explains why t_{dip} and t_H grow exponentially with the system size, which makes the detection of the correlation experimentally challenging. To handle this problem, we deal with small system sizes. The other issue for both quantities is the lack of self-averaging [56–58], which requires the use of ensemble averages for any system size. We use ensembles that are small, although large enough for revealing the ramp.

III. DYNAMICAL MANIFESTATIONS OF MANY-BODY QUANTUM CHAOS

We start the analysis of the correlation hole with the spin-1/2 Heisenberg model and show that dynamical manifestations of many-body quantum chaos can be detected in a chain with only 6 sites. Moving next to the long-range interacting Ising model, we verify that the correlation hole is not as clearly discernible for small system sizes as in the Heisenberg model.

A. Disordered spin-1/2 Heisenberg model

Spin-1/2 Heisenberg models and similar models can be experimentally realized with NMR platforms [52, 53], inelastic neutron scattering [65], cold atoms [17], Rydberg atoms [66], ion traps [23], and quantum dots [67]. In the presence of onsite disorder, this model has been extensively used in studies of many-body localization [68–70].

We consider a 1D isotropic spin-1/2 Heisenberg (XXX) model with nearest-neighbor couplings and open boundary conditions described by the Hamiltonian

$$\hat{H}_{\text{XXX}} = \frac{1}{2} \sum_{k=1}^L h_k \hat{\sigma}_k^z + \frac{J}{4} \sum_{k=1}^{L-1} \vec{\sigma}_k \cdot \vec{\sigma}_{k+1}, \quad (11)$$

where L is the chain size, $\vec{\sigma}_k \equiv \{\hat{\sigma}_k^x, \hat{\sigma}_k^y, \hat{\sigma}_k^z\}$ are the Pauli operators on the k th site, the random Zeeman splittings h_k are uniformly distributed within $[-W, W]$, and the coupling strength $J = 1$. The Hamiltonian conserves the total spin in the z -direction, $\mathcal{S}_z = \sum_k \hat{\sigma}_k^z/2$, so the Hamiltonian matrix consists of $L+1$ mutually decoupled diagonal blocks. We choose L to be even and work in the largest subspace, where $\mathcal{S}_z = 0$ and the Hilbert-space dimension is $D = L!/(L/2)!^2$.

In the absence of disorder, $W = 0$, the XXX model is integrable and solvable via the Bethe ansatz [71]. When the disorder strength $W \sim J$, the system is chaotic, thus presenting correlated eigenvalues [72–74]. For $W \gg J$,

the spectra of finite systems show Poisson statistics [74] suggesting localization.

To study the dynamics, the system is initially far from equilibrium. It is prepared in an eigenstate of $\hat{H}_z = \sum_{k=1}^L h_k \hat{\sigma}_k^z/2 + J \sum_{k=1}^{L-1} \hat{\sigma}_k^z \hat{\sigma}_{k+1}^z/4$, which represents the unperturbed part of the total Hamiltonian \hat{H}_{XXX} . These initial states have on each site a spin pointing up or down in the z -direction, such as the Néel state, $|\uparrow\downarrow\uparrow\downarrow \dots\rangle$, and they can be experimentally prepared. We choose initial states from the middle of the spectrum, $\langle \Psi(0) | \hat{H}_{\text{XXX}} | \Psi(0) \rangle \sim 0$, where chaos is certain to develop.

1. Survival probability

In Fig. 2(a) and Fig. 2(b), we show the evolution of the averaged survival probability for the XXX model in Eq. (11) in the chaotic ($W = 0.5$) and localized ($W = 3$) phase, respectively. The data are averaged over 10 initial states and 50 disorder realizations for the chaotic model and 500 disorder realizations for the localized regime. The solid lines are obtained by further smoothing the data by passing them through a Savitzky-Golay filter [75] using a zero-order polynomial running over 10 consecutive points, which is equivalent to a running time-average. The horizontal dot-dashed lines indicate the saturation value, $\overline{S_P}$. Our goal is to identify a ramp below $\overline{S_P}$ for the system in the chaotic regime.

In Fig. 2(a), we observe that the averaged survival probability exhibits the slope-dip-ramp-plateau structure even for system sizes as small as $L = 6$ and $L = 8$. The structure is particularly visible for $L = 6$, because the system size is so small that the dynamics does not develop a power-law decay. In this case, the Gaussian decay, $e^{-\Gamma^2 t^2}$, is followed by the ramp, which makes the correlation hole rather evident. For $L = 6$, the minimum value of $\langle S_P(t) \rangle$ at t_{dip} is large enough, $\langle S_P(t_{\text{dip}}) \rangle_{\text{min}} \sim \mathcal{O}(10^{-1})$, and t_{dip} is small enough, $t_{\text{dip}} \sim \mathcal{O}(10)$, to be within the grasp of current experimental setups. Even the saturation time, $t_H \lesssim 10^2$, is at the limit of what can be experimentally reached. Notice, however, that it is not essential to run the experiment up to t_H . To convince oneself that chaos has been dynamically captured, it should suffice to detect the ramp, that is, to measure values of $\langle S_P(t) \rangle$ that consistently increase as time passes, following the ramp described by the $b_2(t)$ function in Eq. (3).

The purpose of Fig. 2(b) is to make evident that even though one can find values of $\langle S_P(t) \rangle$ below the saturation line when the system is non-chaotic, these values are not consistently below $\overline{S_P}$ and they are not on a ramp described by $b_2(t)$, so they are not caused by the presence of correlated eigenvalues. We chose to show the non-chaotic case with $W \neq 0$ instead of $W = 0$, because we could smooth the curves using averages over disorder realizations, but the discussion is valid for both cases.

Due to its lack of self-averaging, the survival proba-

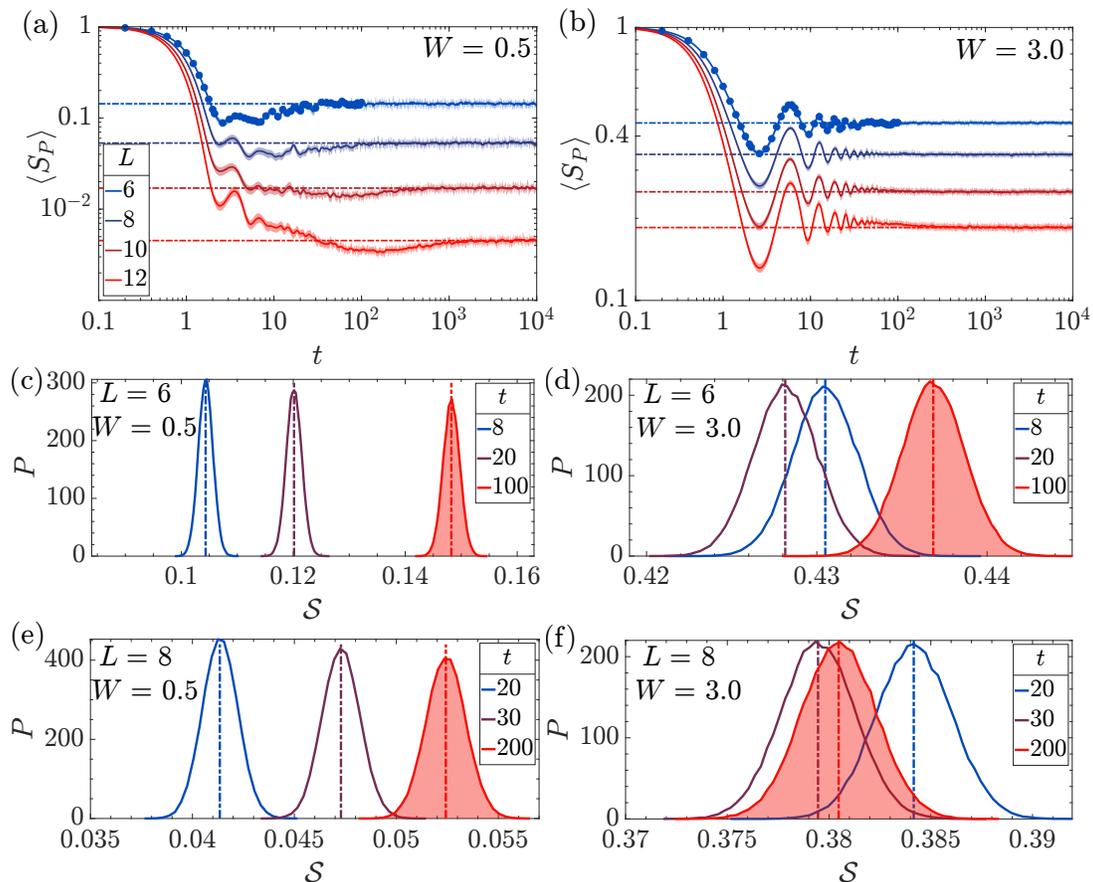


FIG. 2. (a)-(b) Evolution of the survival probability under the Heisenberg spin-1/2 model in Eq. (11) in the (a) chaotic ($W = 0.5$) and (b) localized ($W = 3$) regime for various system sizes L . Average over 10 initial states and 50 (500) disordered realizations for $W = 0.5$ ($W = 3$). All data are further smoothed with the Savitzky-Golay filter [75] resulting in the solid lines. The shaded area around the lines denotes the 95% confidence interval of the ensemble averaged survival probability. Blue dots in (a)-(b) for $L = 6$ indicate the survival probability obtained with the Qiskit statevector simulator, averaged over the same trajectory parameters as the classical simulation. Horizontal dot-dashed lines mark the asymptotic values, $\overline{S_P}$. Time is in the unit of $J = 1$. (c)-(f) Shot noise experiment for the survival probability with $M = 100$ measurements for each initial state (10 samples) and disorder realization (50 samples). Distributions P of S for different system sizes, disorder strengths, and times as indicated in the panels. The shaded distribution is obtained for a time in the saturation region. Vertical lines mark the average values $\langle S \rangle$ for the chosen times.

bility has to be averaged to display the correlation hole. In Fig. 2(a), we have 10 initial states and 50 disorder realizations, but we verified numerically that 10 random realizations suffice to reveal the ramp. A much larger number of realizations are needed for the convergence of the results in the localized phase [Fig. 2(b)] than in the chaotic regime [Fig. 2(a)]. This is because in the chaotic regime, the relative variance of the fluctuations of $\langle S_P(t) \rangle$ at long times remains constant as L increases [56], while it increases with system size in the localized phase [57].

In Fig. 2(a) and Fig. 2(b), for $L = 6$, we compare the results obtained with classical simulations (solid lines) with simulations performed with Qiskit (blue dots) [76]. The agreement is excellent both in the chaotic and localized regimes. The simulations in Qiskit use the statevector backend, which is a noiseless simulator that returns the exact quantum state of the system at each time

step after evolution. The obtained statevector is then compared to the initial state to determine the survival probability. In a real quantum computer the survival probability can be computed with a swap test between the initial and evolved state [77]. The success of the experimental execution on a modern quantum hardware is dependent on the circuit robustness to shot noise, the random fluctuation associated with measurement.

2. Shot-noise experiment

Experimentally, the averaged survival probability is measured as follows. One chooses a particular realization of the onsite disorder, prepares the system in a specific initial state $|\Psi(0)\rangle$, and after letting it evolve unitarily for a time t , one performs a projective measurement.

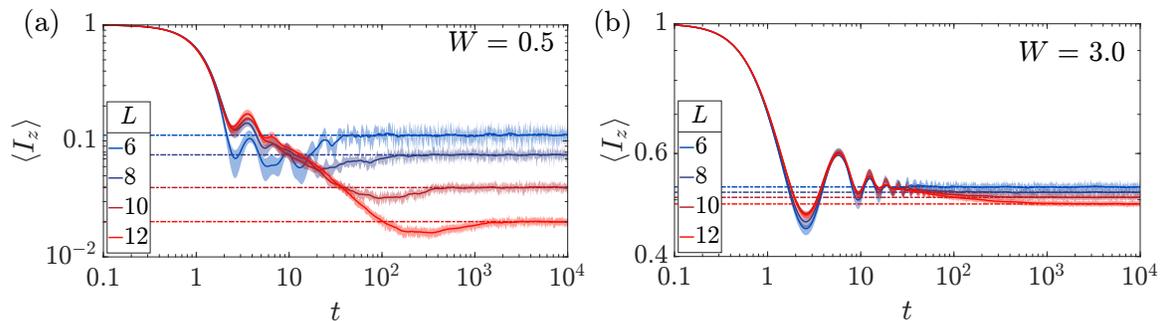


FIG. 3. Evolution of the spin-autocorrelation function under the Heisenberg spin-1/2 model in Eq. (11) in the (a) chaotic ($W = 0.5$) and (b) localized ($W = 3$) regime for various system sizes L . Average and smoothing as in Fig. 2. Time in the unit of $J = 1$. The horizontal dot-dashed lines denote the asymptotic values, \bar{I}_z .

Each measurement corresponds to a “shot” and its outcome can be either 0 or 1 [12]. For M number of shots, one gets the outcome 1 for M_1 number of times, where $0 \leq M_1 \leq M$. According to the measurement postulate of quantum mechanics, $\lim_{M \rightarrow \infty} \frac{M_1}{M} = S_P(t)$, where $S_P(t)$ is the survival probability, as defined in Eq. (5). This procedure is repeated for 10 initial states and 50 random realizations to get the averaged survival probability, $\langle S_P(t) \rangle$.

To emulate the experiment, we define a random variable

$$s = \begin{cases} 1 & \text{with probability } S_P(t) \\ 0 & \text{with probability } 1 - S_P(t) \end{cases}. \quad (12)$$

For each initial state and disorder realization, the sequence of M random numbers s gives a value $\tilde{S}(t) = M_1/M$. We then define $\mathcal{S}(t)$ as the average of $\tilde{S}(t)$ over 10 initial states and 50 disorder realizations. If M is very large, then $\mathcal{S}(t) \rightarrow \langle S_P(t) \rangle$.

For a finite M , every time we repeat the procedure above, we get a value of $\mathcal{S}(t)$ that fluctuates around $\langle S_P(t) \rangle$. This fluctuation is called shot noise and is proportional to $1/\sqrt{M}$. The distribution of the values of $\mathcal{S}(t)$ has a width \mathcal{K}/\sqrt{M} , where \mathcal{K} can be obtained by studying the width of the distribution of $\mathcal{S}(t)$ as function of M .

To identify the correlation hole, the uncertainty in $\mathcal{S}(t)$ must be smaller than the depth of the correlation hole, $\delta = \langle S_P(t_H) \rangle - \langle S_P(t_{\text{dip}}) \rangle$. To resolve $\mathcal{S}(t_{\text{dip}})$ from $\mathcal{S}(t_H)$ with 99.73% certainty, we need to have $\delta > 3\mathcal{K}/\sqrt{M}$. Based on this reasoning, we estimate that $M \sim 40$ for the survival probability evolving under the XXX model with $L = 8$ and even less for $L = 6$. This means that $\mathcal{O}(10^2)$ shots per initial state and realization should be sufficient to separate the minimum of $\langle S_P(t) \rangle$ from its corresponding asymptotic value.

In Figs. 2(c)-(f), we show distributions of the values of $\mathcal{S}(t)$ for $W = 0.5$ [Figs. 2(c),(e)] and $W = 3$ [Figs. 2(d),(f)] for $L = 6$ [Figs. 2(c),(d)] and $L = 8$ [Figs. 2(e),(f)]. We use 10 initial states, 50 disorder realizations, and $M = 100$, and repeat the procedure 10^5

times to obtain the density of $\mathcal{S}(t)$. The distributions are depicted for three different times. Their choices are based on the numerical results for the dynamics for $W = 0.5$, so that the shortest time is close to the point where the ramp starts, the second one is an intermediate time on the ramp, and the largest time is already in the region of the saturation of the dynamics. The distribution for the longest time is shaded.

We see that in the chaotic regime [Figs. 2(c),(e)], the distributions of $\mathcal{S}(t)$ are well separated, so that the values of the survival probability on the ramp can be reliably identified against the asymptotic value, and the averages of the distributions grow monotonically as time increases. In contrast, the distributions of $\mathcal{S}(t)$ for the localized phase [Figs. 2(d),(f)] are not separated. Furthermore, the average $\langle \mathcal{S}(t) \rangle$ does not grow monotonically with time. In Fig. 2(d), $\langle \mathcal{S}(t) \rangle$ for $t = 20$ is smaller than for $t = 8$, and in Fig. 2(f), the shaded distribution, which is for a time already in the saturation region, has $\langle \mathcal{S}(t) \rangle$ smaller than for the chosen intermediate time.

The results in Figs. 2(c),(e) imply that the experimental detection of the correlation hole should be possible with a chain with only $L = 6$ or $L = 8$ sites. It requires approximately 10^4 measurements done at a few selected times in the interval where the ramp develops.

3. Spin autocorrelation function

In Fig. 3(a) and Fig. 3(b), we show the evolution of the averaged spin-autocorrelation function for the XXX model in Eq. (11) in the chaotic ($W = 0.5$) and localized ($W = 3$) phase, respectively. The data are averaged and smoothed as in Fig. 2. Similarly to the case of the survival probability, a correlation hole is visible in the chaotic regime [Fig. 3(a)] even for small system sizes. However, despite the experimental advantage of the spin autocorrelation function as a local quantity, the numerical results are noisier than for the survival probability, because $I_z(t)$ can also have negative values. This means that, compared with the survival probability, a larger number of measurements should be experimentally

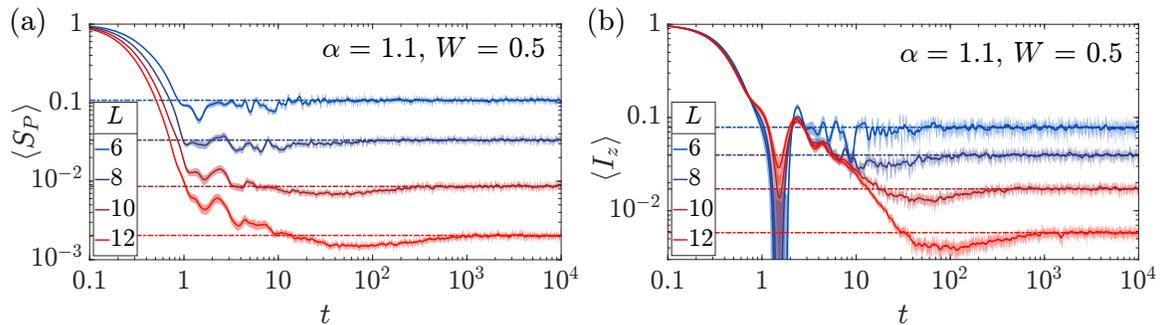


FIG. 4. Evolution of the (a) survival probability and (b) spin-autocorrelation function under the long-range interacting spin-1/2 Ising model in Eq. (13) with $B = 2$, $W = 0.5$, and $\alpha = 1.1$ for various system sizes, L . The data are averaged over 10 initial states and 50 disorder realizations, and are further smoothed with the Savitzky-Golay filter (solid lines), as described in Fig. 2. Time in the unit of $J = 1$. The horizontal dot-dashed lines denote the asymptotic values.

required to reproduce the results in Fig. 3 and to distinguish the ramp from the saturation value.

B. Disordered long-range interacting Ising model

We now analyze a disordered chain of spin-1/2 particles with long-range interaction, as those experimentally realized with ion traps [20–22]. Our model also has onsite disorder and open boundary conditions, being described by the Hamiltonian

$$\hat{H}_{\text{LR}} = \frac{1}{2} \sum_{k=1}^L (B + D_k) \hat{\sigma}_k^z + \sum_{j < k} \frac{J}{(k-j)^\alpha} \hat{\sigma}_j^x \hat{\sigma}_k^x, \quad (13)$$

where B indicates a constant magnetic field in the transverse direction, D_k is uniformly distributed within $[-W, W]$, and α controls the range of the spin-spin interaction. To link with the experiment in [22], we choose $B = 2$ and $\alpha = 1.1$. As in the XXX model, we take $J = 1$ to fix the energy unit and $W = 0.5$ to access the chaotic regime. For $W = B = 0$, \hat{H}_{LR} describes the Sherrington-Kirkpatrick spin glass model [78], while the infinite-range interaction limit ($\alpha = 0$) yields the Lipkin-Meshkov-Glick model, which is an ideal test-bed for phenomena like excited-state quantum phase transition [79–81] and quantum scars [82].

The Hamiltonian \hat{H}_{LR} decomposes into two symmetry sectors, one spanned by spin configurations with an odd number of up-spins in the z -direction and the other with an even number of up-spins. To maximize our access to the center of the spectrum, we use the sector with an even number of up-spins when $L/2$ is even, and the odd sector otherwise. The initial states are once again product states in the z -direction, which are experimentally accessible.

In Fig. 4, we show the evolution of the averaged survival probability [Fig. 4(a)] and the averaged spin-autocorrelation function [Fig. 4(b)] for the long-range interacting Ising model in Eq. (13) in the chaotic regime ($\alpha = 1.1$, $W = 0.5$). In contrast with the Heisenberg

model, the correlation hole for $\langle S_P(t) \rangle$ becomes clearly visible only when $L > 8$ and for $\langle I_z(t) \rangle$, it needs $L \geq 8$, which makes the experimental detection of the correlation hole more challenging for the long-range interacting Ising model.

IV. SINGLE-EXCITATION CASE

To provide one more case in which dynamical manifestations of spectral correlations could be experimentally detected with available capabilities, we resort to a system with a single excitation described by the following Hamiltonian

$$\hat{H} = \frac{1}{2} \sum_{k=1}^L h_k (\hat{\sigma}_k^z + \mathbb{I}) + \frac{J}{4} \sum_{k=1}^{L-1} (\hat{\sigma}_k^x \hat{\sigma}_{k+1}^x + \hat{\sigma}_k^y \hat{\sigma}_{k+1}^y), \quad (14)$$

where h_k 's are random Zeeman splittings uniformly distributed in $[-W, W]$. This Hamiltonian is similar to that in Eq. (11), where only nearest-neighboring couplings are present, but it does not have the Ising interaction. In the thermodynamic limit, the system in Eq. (14) exhibits Anderson localization for any finite disorder strength [83] with a localization length for eigenstates away from the edges of the spectrum given by $l \approx 6.5653/W^2$ [84]. To investigate finite system sizes, one can then take the scaled localization length $\xi = 6.5653/(W^2 L)$ as a parameter.

The system in Eq. (14) is not chaotic, but if the localization length is larger than the system size, the energy levels are correlated and show Wigner-Dyson distribution [85–87]. Despite being just a finite-size effect, these spectral correlations also get dynamically manifested [86] and could then be experimentally detected.

Figure 5(a) provides the results for the analysis of short-range correlations done with the ratio [88, 89],

$$r_n = \min \left(\tilde{r}_n, \frac{1}{\tilde{r}_n} \right), \quad \text{where} \quad \tilde{r}_n = \frac{E_{n+1} - E_n}{E_n - E_{n-1}}. \quad (15)$$

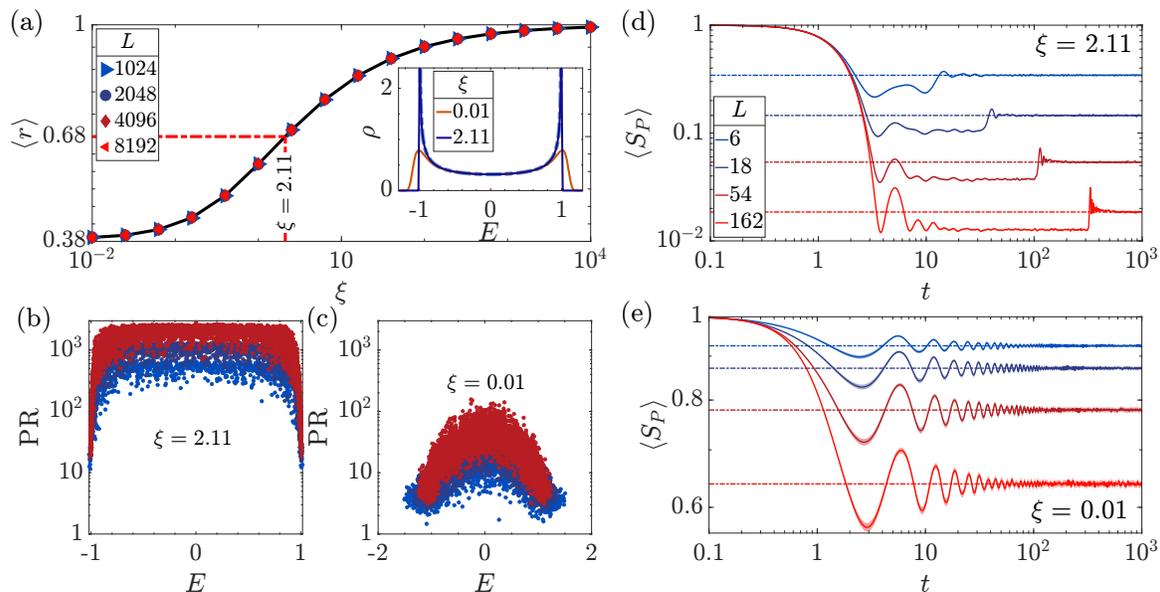


FIG. 5. Results for the spin-1/2 model in Eq. (14) with a single excitation. (a) Average ratio of level spacing, $\langle r \rangle$, as a function of $\xi \propto (W^2 L)^{-1}$ for different system sizes, L . The red dashed line indicates the value $\xi = 2.11$ where $\langle r \rangle \approx 0.68$, which is the average ratio observed in GSE. The inset shows the density of states $\rho(E)$ averaged over 10^4 disorder realizations for two different values of ξ and $L = 8192$; the dashed line indicates the analytical expression of the density of states for $\xi \gg 1$. (b)-(c) Participation ratio as a function of energy from a single disorder realization for (b) $\xi = 2.11$ and (c) $\xi = 0.01$. (d)-(e) Survival probability for (d) $\xi = 2.11$ and (e) $\xi = 0.01$ for different system sizes. The initial state has the excitation on the first site of the chain. The data are averaged over 200 disordered realizations and further smoothed with the Savitzky-Golay filter. Time is in the unit of $J = 1$. The horizontal dot-dashed lines denote the asymptotic values of $\langle S_P(t) \rangle$.

The figure shows the average $\langle r \rangle$ over r_n for states in the middle of the spectrum as a function of ξ . As ξ increases and the localization length becomes larger than the system size, the level spacing distribution moves from Poisson (absence of correlations) to distributions that indicate stronger and stronger level repulsion, reaching statistics equivalent to those for Gaussian orthogonal ensemble (GOE), Gaussian unitary ensemble (GUE), Gaussian symplectic ensemble (GSE), and beyond (picket-fence). When $\xi = 2.11$ [dashed line in Fig. 5(a)], the average ratio $\langle r \rangle$ is approximately that observed in the GSE. This is the value that we use for the analysis of the survival probability. We compare the results with those for $\xi = 0.11$, for which the level spacing distribution is Poissonian.

The comparison of the participation ratio,

$$\text{PR}_n = \frac{1}{\sum_k |\langle k | E_n \rangle|^4}, \quad (16)$$

as a function of E_n for $\xi = 2.11$ [Fig. 5(b)] and $\xi = 0.01$ Fig. 5(c)] further corroborates that the finite system in Eq. (14) is delocalized when ξ is large. Notice that the density of states, shown in the inset of Fig. 5(a), is not Gaussian as in many-body systems, although the most delocalized states are still those in the middle of the spectrum.

To investigate the evolution of the survival probability, we prepare the system in a initial state where the excitation is on the first site of the chain. This is done, because

for $W \rightarrow 0$, the shape of the LDOS for this state is semi-circular [86], which brings us closer to case of Fig. 1 and should facilitate the visibility of the dip below \bar{S}_P .

We show the evolution of the averaged survival probability obtained for $\xi = 2.11$ in Fig. 5(d) and for $\xi = 0.01$ in Fig. 5(e). The behavior in the two panels is completely different. The survival probability in Fig. 5(d) exhibits a “correlation plateau” at $\langle S_P(t) \rangle \sim 2/L$, which is below the saturation plateau at $\bar{S}_P \sim 3/L$ [90], while $\langle S_P(t) \rangle$ in Fig. 5(e) simply oscillates around \bar{S}_P .

The results in Fig. 5(d) indicate that even though the long-time dynamics of the survival probability for the Hamiltonian in Eq. (14) is not described by the $b_2(t)$ function in Eq. (3), as in truly chaotic systems, one can still capture dynamical manifestations of spectral correlations in spin systems with a single excitation. In this case, since the saturation (Heisenberg) time scales linearly with the chain size ($t_H \propto L/\Gamma$, where $\Gamma \sim 1/2$), it should be viable to experimentally run the entire evolution up to saturation. This simple scenario could serve as a first step towards the experimental detection of many-body chaos.

V. CONCLUSION

Experimental advances in engineering pure initial states, preserving quantum coherences for long times,

and monitoring the time evolution of quantities of interest to many-body quantum systems set the stage for the direct observation of dynamical manifestations of many-body quantum chaos, specifically of the onset of the slope-dip-ramp-plateau structure (correlation hole), that is typical of systems with correlated eigenvalues. Several existing quantum simulators and quantum computers manipulate ensembles of coupled qubits, which can be generally described by interacting spin-1/2 models. This motivated our analysis of the emergence of the correlation hole in the survival probability and in the spin autocorrelation function evolved under two chaotic spin-1/2 models: the 1D disordered Heisenberg model with nearest-neighbor couplings and the 1D disordered long-range interacting Ising model in a transverse field.

We concluded that the averaged survival probability evolved under the disordered Heisenberg chain with only 6 sites offers the best prospect for the detection of many-body quantum chaos with available experimental resources. Furthermore, our analysis of the shot-noise experiment for the survival probability indicated that measurements at just a few times within the correlation hole should suffice for inferring its presence.

Due to the small number of qubits and the short Heisenberg time of the 6-site chain, the quantum circuits needed to evolve the state is relatively shallow. Several algorithms, such as Trotterization[91, 92] or hybrid algorithms, are promising candidates for the detection of the correlation hole. For a quantum algorithm to be successful on current hardwares, it will need to balance circuit depth and the precision of the results. For example, if Trotterization is employed, the step size can be increased, sacrificing precision for a lower gate depth. Hybrid approaches to time evolution are also promising, as these algorithms limit the required gate depth through the introduction of a classical computer.

The detection of dynamical manifestations of spectral correlations, but not necessarily many-body quantum chaos, could also be achieved with quantum systems of few excitations or few degrees of freedom. In the presented case of a spin-1/2 model with a single excitation, correlated eigenvalues are due to finite-size effects. They

get manifested in the dynamics at times that are shorter and at values of the survival probability that are larger than what we have for many-body systems of the same length. Another model that could be used for the experimental detection of quantum chaos is the Dicke model, which describes a set of N spin-1/2 particles collectively interacting with a single-mode field. Because the interaction is collective, the system has only two degrees of freedom. In the chaotic regime, a “correlation ramp” emerges in the evolution of the survival probability [45] even when considering initial coherent states [93].

We close this work with a brief discussion about the case of noisy systems. Recent investigations of the spectral form factor in energy dephasing scenarios [8, 10, 12, 13], non-Hermitian Hamiltonians [9, 11], and parametric quantum channel models [13] suggest that weak interactions with the environment reduce the need for ensemble averaging. This implies that some noise can be beneficial for the experimental detection of the correlation hole.

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